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# COMPARISON OF PROPAGATION PREDICTION MODELS WITH SUDS I ACOUSTIC DATA

by

Halcyon E. Morris

Undersea Surveillance and Ocean Sciences Department

April 1974

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**ADMINISTRATIVE INFORMATION**

The Surface Duct Sonar Measurements (SUDS) Phase B Program was sponsored by the Office of Naval Research, Code 102-OSC, CDR T. J. McCloskey (Contract N00014-72-C-0115). The principal objective was to compare various propagation prediction models with measured data obtained on the SUDS I surface duct experiment. The surface duct experiment, Phase A, was primarily sponsored by the Naval Ship Systems Command, Sonar Technology Division, PMS-302-4 (Task 16127), and partly sponsored by the Office of Naval Research, Code 102-OSC (Contract N00014-72-C-0115).

The work in this report was performed from 1 July 1972 to 1 July 1973 by members of the Undersea Surveillance and Ocean Sciences Department. The report was reviewed for technical accuracy by Dr. Homer P. Bucker.

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reasonable predictions of sound propagation in the surface duct. Modified AMOS gave predictions in and below the duct, while ASRAP II was used only for in-duct predictions. The Virtual Mode Model needs the addition of the rough surface corrections that are part of the Natural Mode Model. The Natural Mode WAVE Model with rough surface corrections provided somewhat better predictions than the semiempirical propagation models.

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## SUMMARY

### PROBLEM

Compare recently developed theoretical models for sound propagation loss in the ocean with the semiempirical fleet prediction models and experimental measurements from the Surface Duct Sonar Program (SUDS).

### RESULTS

The environmental data sets containing averaged surface duct parameters have been completed. Extensive environmental data collected during the experiment showed the SUDS I tests to be in an area of weak and variable ducts. Representative comparisons are shown for two operational fleet prediction models and two laboratory theoretical models for five different areas. The main conclusions from the propagation loss versus range values are as follows.

1. Modified Acoustic, Meteorological, and Oceanographic Survey (AMOS) and Acoustic Sensor Range Prediction Model (ASRAP II) provided reasonable predictions of sound propagation in the surface duct. Modified AMOS gave predictions in and below the duct, while ASRAP II was used only for in-duct predictions.
2. The Virtual Mode Model needs the addition of the rough surface corrections that are part of the Natural Mode Model.
3. The NUC Rough Surface Natural Mode Model (WAVE) with surface corrections provided somewhat better predictions than the semiempirical propagation models. However, because of the much greater complexity of the wave model, it may not be suitable for fleet predictions at the present time.

### RECOMMENDATIONS

1. Continue analysis of the SUDS I environmental and acoustic data. Further analysis of variable duct cases is needed to develop the changing profile propagation models that can be reasonably expected to be in use by the fleet in the next decade.
2. Conduct one or two more sea tests. Continuing sea tests at higher frequencies and in warmer waters are needed for evaluation of new surface duct sonars. Secondly, at the other end of the frequency spectrum, there are several proposed systems that operate at lower frequencies than were covered by the SUDS I data set. Finally, the accuracy of the principal fleet prediction model (modified AMOS) has not been tested when the ducts have strong sound-speed gradients. Continuation of the work will confirm validation of the fleet models over a wider range of duct conditions, as well as provide the experimental data required for development of variable profile models.

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## INTRODUCTION

The Surface Duct Sonar (SUDS) Program was undertaken in the Undersea Surveillance and Ocean Sciences Department at the Naval Undersea Center to collect surface duct propagation data sets and to test the accuracy of current semiempirical fleet prediction models against these data. An additional scientific objective was to test recently developed theoretical models with the semiempirical fleet prediction models and the SUDS data sets. The theoretical models are needed to predict the spatial variations of acoustic field in a surface duct, especially in acoustic warfare problems.

A new set of surface duct data was needed because of requirements for sonar performance predictions below 5.0 kHz, which are used in the employment of fleet sonar units and sonobuoy systems. Also, new thermistor buoys that allow large-scale synoptic sampling of sound speeds throughout the duct and new surface roughness measuring devices that determine sea surface-wave spectra have become available. Previously available data sets were from the Acoustic, Meteorological, and Oceanographic Survey (AMOS) tests, which were conducted by the Naval Underwater Sound Laboratory more than 20 years ago (reference 1). While the AMOS data continue to be useful, there are very little data in the set below 8.0 kHz, and the concomitant environment data are sparse. The Navy Electronics Laboratory (NEL) collected a limited set of surface duct data for a well defined duct in 1955. This experiment was run during calm sea conditions at two hydrophone depths and two frequencies. The results have been reported by Pedersen and Gordon (reference 2), who showed close agreement between the data and calculations derived from a natural mode solution of the wave equation. In 1964, NEL manned a second surface duct experiment which was reported by Arthur D. Little, Inc. (reference 3). This was also limited in scope, and there were problems with the quality of the experimental data. To generate low-frequency surface duct data over a wider range of frequencies and during more varied environmental conditions than had been encountered in the NEL tests, NAVSHIPS sponsored a large sea test conducted by Lockheed California Company (reference 4). Many problems plagued a new digital recording system built for this test, and there are serious questions about the validity of the collected data (reference 5). The most recent surface duct experiment was SUDS. For low frequencies (the CW data ranged from 0.4 to 5.0 kHz), this set is the only complete one, and SUDS is also unique in containing an adequate sample of related environmental data.

The detailed sets of measurements, hereafter referred to as SUDS I, were collected from 7 to 25 February 1972. The measurements involved three ships (USNS DeSTEIGUER, USNS S. P. LEE, and NUC R/V CAPE) as illustrated in figure 1\*. The ships occupied four different stations, approximately 300 miles southwest of San Diego, California (figure 2). Propagation loss with a CW pulse was measured at several frequencies, in particular at 0.4, 1.0, 1.5, 2.5, 3.5, and 5.0 kHz, with concurrent sound-speed profiles along the

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\*Figures are at the end of this report.



propagation paths. Preliminary results from the SUDS I cruise are reported in reference 6, which describes the experimental procedures and measurements. In addition, the complete acoustic data set is presented in plots of propagation loss versus range. Much of the material in this introduction has been extracted from reference 6.

Presentation of the SUDS data analyses is continued in this report. Results from two operational models and three laboratory models that predict propagation loss are compared with the SUDS I acoustic data set. The results are evaluated, and conclusions and recommendations are presented.

Evaluation of prediction models for the surface duct mode is important to the Navy because most submarine detections — whether active or passive, by ship, submarine, airplane, or helicopter — are made via direct path or surface duct sound propagation.

Most operational commanders have found that submarines spend much time near the surface during ASW operations and exercises. Conventional submarines must be near the surface to snorkel and are, therefore, especially vulnerable to detection. Nuclear submarines also spend considerable time near the surface. Although less likely to be detected while deep, they themselves are limited acoustically below the surface layer.

While convergence zone detection might be possible over a large portion of a given ocean, ASW operations may not be spread evenly, but concentrated closer to land, where convergence zone propagation does not exist. Also, ocean depths of nearly 3000 fathoms are required for convergence zones in most tropical and semitropical areas.

The development of long range sonar systems does not eliminate the need for direct path or surface duct systems. Both approaches complement each other, and both are indispensable. In reality, all active sonars now in service detect primarily by propagation involving the surface duct and related paths. Classification and attack especially rely on direct path propagation.

High-frequency and localization sonars are definitely limited in range, and propagation in the mixed layer is a major operational mode. Submarine sonars are not used in the active mode for search, but are usually keyed for ranging information at short ranges within weapon capability. Hence, the surface duct mode is highly important for keyed active operations. Passive detections are also commonly attained by means of the surface duct for submarines, sonobuoys, and towed arrays. Thus, the mode is significant for present and future passive systems.

## EXPERIMENTAL PROCEDURES AND MEASUREMENTS

The SUDS I experiment is described in detail in the scientific plan for SUDS I (reference 7), which was distributed prior to the cruise. Three ships were involved in the operations (figure 1). The USNS LEE towed the thermistor chain and projectors. Sound-speed profiles were continuously sampled by the towed thermistor chain every 15.2 meters along the track. Projector frequencies were 0.4, 1.0, 1.5, 2.5, 3.5, and 5.0 kHz. Explosives for frequencies from 0.4 to 10 kHz were used to investigate low-frequency cutoff characteristics of the duct, provide broadband coverage, and validate results of explosives as compared to projectors. Preliminary results were reported in the status report for SUDS I (reference 8), distributed about 5 weeks after the cruise. Additional results were reported by Keir and Townsen (reference 9).



The USNS DeSTEIGUER served as the receiving ship for the acoustic signals. A string of hydrophones was spaced so that three hydrophones were in the mixed layer and two were below the layer at 1.3 and 2.0 times the layer depth. Propagation through the thermocline and leakage out of the duct was therefore included.

The R/V CAPE operated a thermistor buoy system that sampled sound-speed parameters every 10 seconds along the propagation path. In addition to the thermistor chain and thermistor buoy data, expendable bathythermographs (XBT's) and some salinity, temperature, depth, and sound velocity (STD-SV) data were taken. Wave height data were collected by a spar buoy and Datawell Waverider.

Figure 2 illustrates the areas occupied during the measurements. The stations were selected on the basis of layer depths in locations where the water depth was about 2000 fathoms or more. The layer depth was 40 to 70 meters on Stations 1 and 2 and 80 to 100 meters on Stations 3 and 4. Deep water was required for adequate separation of surface duct and bottom bounce arrivals.

At Station 1, because of irregular currents, the drift rate was small and erratic despite light winds from three directions, each lasting at least 24 hours. Station 2 was characterized by 18-knot northerly winds which account for the shown track. Light northerly winds, encountered on Stations 3 and 4, generally account for the observed drift.

The acoustical measurements are of good quality, complete, and comprehensive. The measurements include propagation loss at CW frequencies from 0.4 to 5 kHz, ranges from 0 to 18 miles, and receiver depths from 4 to 180 meters. The data are in digital format on computer cards and magnetic tape.

## SURFACE DUCT PROPAGATION AND PREDICTION MODELS

The important features of surface duct propagation are shown in figure 3. If the surface layer is not sufficiently mixed (figure 3a), the sound-speed gradient is negative, and no surface duct exists. In figure 3b, there is a mixed layer in which the water temperature is nearly constant, resulting in a slight positive sound-speed gradient because of pressure effects (for isothermal water the sound speed gradient is  $\sim 0.018/\text{sec}$ ). In this case a surface duct exists.

Typical ray paths from a source in the surface duct are shown in figure 3b. Some rays are trapped in the duct, while those with grazing angles at the source greater than  $\arccos(C_s/C_t)$  leak-out of the duct ( $C_s$  and  $C_t$  are the sound speeds at the source and at the bottom of the duct). If a leakage ray is downgoing at the source, it will pass directly through the thermocline, while upgoing rays will reflect from the surface and then pass through the thermocline. The depth of maximum sound speed is called the duct depth. Below this depth lies the thermocline, a region where temperature (hence sound speed) decreases rapidly with depth. In the no duct case, figure 3a, S is the limiting ray beyond which there is a shadow zone (SZ). In figure 3b, T is the limiting ray that defines the shadow zone below the duct, while there is no SZ in the duct.

If the surface of the duct is smooth, then ray theory predicts zero sound-pressure level in the shadow zone. However, the more accurate wave theory predicts diffraction of sound energy below the duct. In both ray and wave theories, there will be a scattering at a rough surface that will partially illuminate the shadow zone.

In general, ray theory does not work well for surface duct calculations. First, there are the diffraction effects, noted above, which result in the calculated sound levels being too high in the duct and too low below the duct. Second, ray theory predicts caustics in the duct which are lines of infinite sound intensity where neighboring rays cross. In principle, there are caustic corrections that smooth out the caustics, but the procedure is difficult and has not been accomplished on a routine basis.

The normal mode type of wave solution seems well suited for ducts without significant horizontal changes. Advantages of utilizing wave theory models are that results should prove more accurate and that models can be better extended into ocean regions where sound-speed profiles are known, but where experimental acoustic data do not exist. The wave theory solution for propagation in a duct was first expressed by Furry (reference 10) for radio wave propagation. Marsh (reference 11) adapted the solution to a surface duct in the ocean. Pedersen and Gordon (reference 2) corrected several errors in Marsh's work and developed practical computer programs for wave solutions. Recently, Bucker (reference 12) extended the theory so that first- and second-order surface scattering effects are included in the solution: loss of energy at the surface and scattering out of the duct, respectively. SUDS I data are used to validate wave models such as this extended wave theory.

To complete the scientific objectives of SUDS I, the propagation data sets were compared with two operational models, modified AMOS used in the Navy Interim Surface Ship Prediction Model (NISSM) and the Acoustic Sensor Range Prediction Model (ASRAP II) used in RP-70. The RP-70 is a set of propagation models developed by the Fleet Numerical Weather Central (FNWC), in cooperation with the Long Range Acoustic Propagation Project (LRAPP). The three laboratory models compared were the Rough Surface Natural Mode (Naval Undersea Center), Ray Mode II (Naval Underwater Sound Center, New London), and Virtual Mode (Bell Telephone Laboratories). A summary of these models is presented in the remainder of this section.

#### OPERATIONAL MODELS

1. Modified AMOS (NISSM)
2. ASRAP II (FNWC)

#### LABORATORY MODELS

1. Rough Surface Natural Mode (NUC)
2. Ray Mode II (NUSC)
3. Virtual Mode (BTL)

Modified AMOS is a semiempirical surface duct prediction model that was first developed at the Navy Underwater Sound Laboratory, New London (reference 1). The modifications to AMOS as used in the Navy Interim Surface Ship Sonar Prediction Model (NISSM) (reference 13) and in the calculations shown in this report consist of a slightly modified volume attenuation coefficient, a modified surface scattering loss, and the addition of a surface duct cutoff term. In the original version of AMOS, the propagation loss is independent of the sound-speed gradient in the duct. However, in the modified version both the surface scattering term and the cutoff term are gradient dependent. The scattering term is proportional to the number of surface reflections of the ray that has a grazing angle of 0 degrees

at the bottom of the duct. As the sound-speed gradient increases, there will be more surface reflections and the propagation loss becomes greater. The surface duct cutoff term is an approximation to the attenuation of the first natural mode. As the gradient increases, the mode attenuation and the propagation loss are reduced. Reference will be made later to the insensitivity of the modified AMOS Model to the strength, i.e., sound-speed gradient, of the duct.

ASRAP II (references 14 and 15) assumes a source near the surface and evenly distributed sound energy throughout the duct which can be calculated by ray theory. To this ray theory framework, additional losses are added for volume attenuation, surface scattering, and duct leakage. Performance predictions below the duct and beyond the limiting ray cannot be made using a ray theory format. In some versions of ASRAP II, the propagation loss at the below-duct receivers is arbitrarily set at a 10-dB greater loss than for in-duct receivers. Because of the lack of a reasonable basis for this assumption, ASRAP II was not used for receiver depths below the duct in this study.

The wave theory models originate with a basic line integral\* that represents the summation of cylindrical waves with different wave numbers. In Ray Mode II (NUSC), an expansion of a factor in the integrand leads to a sum of integrals which can be individually associated with particular ray paths (see reference 16). In Virtual Mode (BTL), a mathematical transformation is made so that the integral form is similar to a classical problem discussed by Titchmarsh (reference 17). Rough surface corrections are not in the current version of this model (reference 18). The Rough Surface Natural Mode Model (NUC WAVE) is basically the Pedersen-Gordon (reference 2) smooth surface theory with rough surface corrections developed by Bucker (reference 12). In this natural model solution, the basic line integral is transformed into two contour integrals, and the solution is calculated as a sum of residues. This model is designated as WAVE in later discussions and graphs.

## COMPARISON OF PREDICTION MODELS WITH ACOUSTIC DATA

### SOUND-SPEED PROFILES

The SUDS I experiment was carried out in an operating area 27° to 30°N latitude and 120° to 124°W longitude, figure 2. This area is in the southwestern edge of the California Current, which is a time- and space-dependent body of water containing periodic diurnal and seasonal variations extending down to approximately 200 to 300 meters. The California Current overlaps another water mass with aperiodic variations extending down to 1200 to 1400 meters. During the experiments, the ducts were weak and variable. During some propagation runs, there was no ducting over parts of the propagation paths.

The prediction models are dependent upon environmental parameters. The models require sound-speed profiles, or surface velocities, and layer gradients. Sound-speed profiles were calculated at frequent intervals over the range of each run. From this data set, a representative profile was selected for each run to be used in the theoretical models.

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\*Private communication with H. P. Bucker. October 1971.

Because of the extremely large amount of SUDS I data, five CW propagation runs (of the possible total of 15) were selected for detailed analysis. To analyze propagation for widely varying environmental conditions, three runs in relatively stable ducts and two runs in unstable ducts were analyzed. The propagation loss in the stable ducts had the usual increase with range, while there was often better propagation at longer ranges than at intermediate ranges in the unstable ducts.

The sound-speed profile for each of the five runs discussed in this report is illustrated and tabulated in figure 4. Each of these averaged sound-speed profiles has a weak positive duct. The layer depth was approximately 50 meters on Station 2 and 80 to 115 meters on Stations 3 and 4.

## DATA ANALYSES

A summary of SUDS I propagation measurements as to type of sound and frequency involved appears in table 1. The shaded areas indicate the runs discussed in this report. The CW propagation runs in SUDS I covered frequencies from 0.4 to 5 kHz. The selected analysis comparisons were made for two runs at 1.5, 2.5, 3.5, and 5.0 kHz and for a single run at 0.4 and 1.0 kHz.

Table 1. Summary of SUDS I Propagation Measurements.\*

Run	Station 1	Station 2	Station 3	Station 4
1	CW PULSE 1.5 kHz 2.5 kHz	CW PULSE 1.5 kHz 2.5 kHz	EXPLOSIVE 0.4 to 10.0 kHz	CW PULSE 3.5 kHz 5.0 kHz
2	CW PULSE 0.4 kHz 1.0 kHz	CW PULSE 1.0 kHz	CW PULSE 1.5 kHz 2.5 kHz	EXPLOSIVE 0.4 to 20.0 kHz
3	CW PULSE 3.5 kHz 5.0 kHz	CW PULSE 3.5 kHz 5.0 kHz	CW PULSE 0.4 kHz 1.0 kHz Variable	CW PULSE 1.5 kHz 2.5 kHz
4	CW PULSE 3.5 kHz 5.0 kHz	CW PULSE 0.4 kHz 1.0 kHz	CW PULSE 3.5 kHz 5.0 kHz	CW PULSE 1.5 kHz 2.5 kHz Variable
5	EXPLOSIVE 0.4 to 10.0 kHz	None	CW PULSE 3.5 kHz 5.0 kHz	None

\* Shaded areas denote runs discussed in the report.

A series of comparisons between the prediction models and the acoustic experimental data is shown. Propagation loss on the plots is defined in decibels as  $-10 \log_{10}$  of the intensity at 1 yard from the source. Modified AMOS is designated AMOS on the plots, and ASRAP II is shortened to ASRAP.

The form of Ray Mode involved a solution based on a random phase summation. The Ray Mode Model was being tested for possible use in the Towed Array Surveillance Systems Range Prediction Program (TASSRAP). This version, however, did not fit the experimental data as well as the other models. As a result, all environmental data were sent to G. A. Leibiger, the developer of the Ray Mode Program, who ran the data with a more sophisticated version of Ray Mode II. The results for one run, Station 2, Run 3, were returned, and examples at two hydrophone depths are included in this report.

The results designated WAVE are from the NUC Rough Surface Natural Mode Model. The NUC Model and the Virtual Mode Model (BTL) are both based on normal mode solutions. The basic differences between the two models are the rough surface corrections which are contained only in the WAVE Model. As a result, the WAVE Model gives a better fit to the data and is shown on the graphs. Examples of comparisons between the WAVE Model without surface corrections and the Virtual Mode Model are shown for one run (figure 6). The calculations for the latter model were done by the Acoustic Environmental Support Detachment (AESP).

Of the five runs selected for analysis (figure 4) not all hydrophone depths for all frequencies are shown in this report. To keep this report at a reasonable size, only examples of each are included. Table 2 lists the frequency and receiver depths for each run. Additional reports will be published for the purpose of showing complete comparisons between experimental data and prediction models.

Table 2. Summary of Frequencies and Receiver Depths Discussed for Each Run in this Report.

Station	Run	Frequency, kHz	Source Depth, m	Receiver Depths, m				
				6	34 to 43	72 to 73	112 to 117	180
3	4	3.5	44	x	x	x	x	x
		5.0	47		x		x	
3	2	1.5	42		x			
		2.5	42			x		
3	3	0.4	42		x		x	
		1.0	42	x				
4	4	1.5	43				x	
		2.5	43					x
2	3	3.5	38		x	x		
		5.0	41					



### Station 3, Run 4

Propagation losses as a function of range are shown for Station 3, Run 4, in figure 5. Results are shown for five receiver depths at 3.5 kHz (parts A through E) and for two receiver depths at 5.0 kHz (parts F and G). Three prediction models — modified AMOS (small dashed line), ASRAP II (large dashed line), and WAVE (solid line) — are compared with the acoustic data. Solid black dots represent one-minute averages of the experimental data. The receiver depths of 6, 36, and 72 meters are in the duct, and the depths of 117 and 180 meters are below the duct. The ASRAP II Model was not used for the hydrophones at 117 and 180 meters because the instruments were below the duct. All models fit this run reasonably well at both frequencies and at the five receiver depths.

Comparisons of Virtual Mode, a model that does not have surface scattering corrections, and a modified version of the WAVE Model, without surface scattering corrections, are shown for Station 3, Run 4, at 3.5 kHz in figure 6. Both models are based on normal mode solutions and should provide similar results. However, there are some discrepancies. When the receiver is below the duct, the results for the Virtual Model deviate from results for the WAVE Model and for the experimental data at ranges less than 8 kyd.

Additional computations were made to test the sensitivity of the AMOS, ASRAP, and WAVE Models to gradient change. The gradient for Station 3, Run 4, was increased from 0.009 to 0.018, the accepted value for an isothermal condition. For the AMOS Model, the propagation loss was altered only tenths of a decibel, and the difference was not distinguishable on the plots for propagation loss. Results are shown for receiver depths of 6 and 180 meters in figure 7. (These results are identical to those in figure 5, parts A and E.) Figure 8 shows the results for the WAVE Model with a gradient of 0.018. When compared with the 0.009 gradient, these calculations range from a greater calculated loss for the gradient of 0.018 at the shallow receiver (6 meters) to a smaller loss for the gradient of 0.018 at a below-duct receiver depth (117 meters). Thus, increasing the gradient by a factor of two in the AMOS Model has essentially no effect on the propagation loss (figure 7), while large effects can result in the WAVE Model (figure 8). For a receiver depth of 6 meters, the ASRAP Model shows less loss for the gradient equal to 0.018 than for the 0.009 gradient (figure 9). The difference in the propagation loss of approximately 2 dB is not significant.

### Station 3, Run 2

Two examples of propagation in the duct are shown for this station in figure 10. At 1.5 kHz, the receiver depth is 37 meters, and at 2.5 kHz, the receiver depth is 73 meters. The AMOS Model predicts somewhat too much propagation loss for this run.

### Station 3, Run 3

Station 3, Run 3, was selected for analysis because of the unstable run conditions. At a receiver depth of 34 meters at 0.4 kHz (figure 11A), the AMOS Model is in the region of the experimental data and follows the trend of the data to a range of approximately 12 kyd. Calculations using the ASRAP II and WAVE Models predict too little loss. However, the agreement between the AMOS Model and the experimental data is probably coincidental because at two other stations, when a 0.4-kHz frequency was used, the AMOS Model

did not fit (reference 6). At a receiver depth of 112 meters, below the duct, the WAVE Model fit the data except for a region of 8 to 16 kyd (figure 11B), and the AMOS Model deviated from the data at 14 kyd and predicted excessive loss at the higher ranges. Results are also shown (figure 11C) for a receiver depth of 6 meters at 1.0 kHz.

#### Station 4, Run 4

This station was also selected because of unstable run conditions. For receivers in the duct, the agreement between the prediction models and acoustic data was not good. Good agreement, however, is shown below the duct at 117 meters at 1.5 kHz and at 180 meters at 2.5 kHz (figure 12).

#### Station 2, Run 3

The temperature data along the propagation run indicated a continuing stable duct situation, much more so than for the other runs discussed in this report. Agreement between the prediction models and experimental data is good (figure 13). The duct was approximately 50 meters so that the results shown for a receiver depth of 43 meters are in the duct, and the results for a receiver depth of 72 meters are below the duct.

Examples of the Ray Mode II Model results for this station are shown for two hydrophone depths. The results agree with the data at 4 meters (figure 14A), but do not agree with the data from the receiver below the duct at 72 meters (figure 14B). These results were anticipated since the model may not work well in the shadow zones where there are no real rays at the receiving points.

### STATISTICAL TESTS

Statistical tests based on differences of propagation loss between the experimental data and the theory were calculated for all theoretical models. However, the natural mode models with their beat patterns do not lend themselves to statistical tests based on differences. The statistical tests included the mean of absolute errors which is a simple measure of the fit in decibels; the quadratic difference function which is a measure of the uniformity of the data; and the linear correlation coefficient which is a measure of the trend of the data. To determine the goodness of fit, it is necessary to consider all the tests. For example, a theoretical curve could have the same trend as the data, but be displaced several decibels which would be revealed with a high mean error.

The equations for the statistical tests are as follows.

$$\text{Difference} = \text{Experimental Data} - \text{Theory} = d_i \quad (1)$$

$$\text{Mean Error} = \frac{1}{N} \sum_{i=1}^N |d_i| \quad (2)$$



$$\text{Average Error} = \frac{1}{N} \sum_{i=1}^N d_i \quad (3)$$

$$\text{Quadratic Difference} = \left[ \frac{\sum [|d_i| - \text{mean error}]^2}{N} \right]^{1/2} \quad (4)$$

$$\text{Linear Correlation Coefficient} = \frac{N \sum X Y - \sum X \sum Y}{[N \sum X^2 - (\sum X)^2]^{1/2} [N \sum Y^2 - (\sum Y)^2]^{1/2}} \quad (5)$$

Three-dimensional plots of these statistical measures were developed so that data at all five receiver depths in 6-kyd range increments could be displayed on a single plot. Examples of three-dimensional plots are shown for Station 3, Run 4, at 3.5 kHz in figure 15. The mean error, average error, quadratic difference, and linear correlation coefficient are for the modified AMOS Model. These results can be compared with similar plots for the ASRAP II Model (figure 16). By a visual comparison of the three-dimensional plots, it can be seen that the ASRAP II and AMOS Models have about the same magnitude of errors for Station 3, Run 4.

Table 3 compares statistical tests for the three receivers in the duct for modified AMOS and ASRAP II Models. Both frequencies for the five runs analyzed are included. The average values suggest that the results are similar and that one model cannot be designated better than the other. The averaged values in decibels for the mean error are 7.1 for AMOS, 6.9 for ASRAP II; average error is -1.1 for AMOS, 2.1 for ASRAP II; quadratic difference is 2.7 for both models; and linear correlation coefficient is 0.84 for AMOS, and 0.83 for ASRAP II.

## DISCUSSION OF RESULTS

At the present time, the environmental data sets containing averaged surface duct parameters have been completed. Data available include averaged sound-speed profiles and surface roughness for each SUDS I run. The acoustic data have been completely reduced so that propagation loss versus range values are available for all SUDS I runs in a format suitable for convenient use by other researchers.

The fleet prediction models, modified AMOS and ASRAP II, provide reasonable predictions for the weak surface ducts encountered during the SUDS I test. As shown earlier in this report, doubling the gradient in the AMOS Model has essentially no effect on the propagation loss, while in the WAVE Model there can be large effects for cases studied in this test. Therefore, the use of AMOS for strong ducts cannot be validated by the SUDS I data set.

The Rough Surface Natural Mode Theory (WAVE) gives somewhat better predictions than the operational models. Subjective analysis by experienced acousticians indicated that of the 50 cases approximately 24 gave the same closeness of fit for either AMOS or WAVE. In approximately 21 cases, the WAVE Model was determined the best fit, compared to six cases for the AMOS Model as the best fit. However, this difference may not be

Table 3. Summary of Statistical Tests for Modified AMOS and ASRAP II for Three Receivers in Duct.

Station	Run	Frequency, kHz		Prediction Model	Receiver Number	Mean Error, dB		Average Error, dB		Quadratic Difference, dB		Linear Correlation Coefficient					
		1	2			Frequency 1	Frequency 2	Frequency 1	Frequency 2	Frequency 1	Frequency 2	Frequency 1	Frequency 2				
2	3	3.5	5.0	AMOS	1	2.8	6.7	-2.0	6.7	1.7	2.1	0.976	0.987				
					2	3.1	5.1	-2.6	5.0	1.7	2.0	0.981	0.987				
					3	4.5	10.8	3.7	10.8	2.1	2.7	0.960	0.975				
3	2	1.5	2.5	ASRAP II	1	4.9	3.8	-3.0	3.3	2.1	2.1	0.966	0.982				
					2	4.7	3.1	-3.3	1.5	1.9	1.8	0.972	0.984				
					3	5.7	10.1	4.9	9.9	2.4	3.2	0.942	0.952				
				AMOS	1	5.7	6.8	-5.0	-4.7	3.2	3.7	0.759	0.392				
					2	10.3	8.0	-10.2	-7.3	3.3	3.1	0.762	0.704				
					3	11.6	5.8	-11.5	-4.5	3.2	2.5	0.740	0.758				
3	3	0.4	1.0	ASRAP II	1	3.8	6.3	-0.5	-1.5	2.5	3.7	0.739	0.368				
					2	5.9	5.4	-5.5	-4.1	2.7	2.6	0.852	0.814				
					3	6.6	3.8	-5.9	-0.2	2.7	2.4	0.683	0.765				
				AMOS	1	10.3	4.4	9.0	-0.8	2.7	2.4	0.805	0.818				
					2	6.8	7.1	-4.6	-6.2	1.8	2.8	0.841	0.745				
					3	10.8	7.3	-9.6	-6.5	1.3	2.5	0.641	0.765				
3	4	3.5	5.0	ASRAP II	1	19.9	5.9	19.9	4.1	2.9	2.9	0.764	0.715				
					2	11.3	5.1	10.5	-1.4	2.1	2.7	0.778	0.638				
					3	6.8	5.3	6.6	-0.5	1.6	2.7	0.533	0.642				
				AMOS	1	5.7	4.7	-5.1	-3.2	2.5	2.5	0.887	0.885				
					2	7.9	4.6	-7.2	-2.9	3.1	2.7	0.849	0.883				
					3	6.1	4.2	-2.9	1.2	2.7	2.6	0.920	0.930				
4	4	1.5	2.5	ASRAP II	1	5.2	5.6	-2.9	-3.3	2.5	3.1	0.868	0.873				
					2	6.0	5.3	-5.0	-3.0	2.8	3.0	0.908	0.887				
					3	4.1	3.8	0.6	2.3	2.5	2.4	0.962	0.942				
				AMOS	1	9.0	12.2	8.9	12.2	2.7	2.6	0.763	0.905				
					2	9.1	8.3	2.3	-1.3	3.0	3.2	0.892	0.878				
					3	7.3	7.3	2.6	1.3	3.9	3.8	0.890	0.875				
4	4	1.5	2.5	ASRAP II	1	12.3	13.2	12.3	13.2	2.9	2.6	0.829	0.840				
					2	8.8	7.3	5.2	-0.2	2.9	3.1	0.957	0.890				
					3	8.5	7.2	6.5	3.6	3.7	3.5	0.883	0.856				
				Average Values } AMOS ASRAP II						Mean Error, dB		Average Error, dB		Quadratic Difference, dB		Linear Correlation Coefficient	
										7.14 6.86		-1.15 2.14		2.67 2.67		0.84 0.83	

significant considering the much greater complexity and consequent higher computational cost of the WAVE Model. The Virtual Mode Model needs the addition of the rough surface corrections that are part of the NUC Natural Mode Model. The Ray Mode Model does not perform as well as the others for the surface duct case and was not included in the final graphs for this report.

## CONCLUSIONS AND RECOMMENDATIONS

The SUDS I tests were in an area of weak and variable ducts. Under these conditions, both fleet prediction models, modified AMOS and ASRAP II, provided reasonable predictions of sound propagation in the surface duct. The modified AMOS program gave predictions in and below the duct, while ASRAP II was used only for in-duct predictions. The Natural Mode Model with rough surface corrections provided somewhat better predictions than the semiempirical propagation models. However, because of the much greater complexity of the WAVE Model it may not be suitable for fleet production at the present time.

It is recommended that the SUDS program be continued, including further analysis of the SUDS I data and the execution of one or two other sea tests. Further analysis of the variable duct cases is needed to develop the changing profile propagation models that can be reasonably expected to be in use by the fleet in the 1980 to 1990 time frame. Continuing sea tests are needed for two principal reasons. First, surface duct data at higher frequencies and in warmer waters are needed for the new surface duct sonar (code name Mallooka) being developed by the Australian Navy and by the United States for small ships of the patrol frigate type. A joint U.S.-Australian experiment has been proposed as a cooperative program. Second, there are several proposed systems, mostly sonobuoy configurations, that operate at lower frequencies than were covered by the SUDS I CW data set.

In summary, analysis of the SUDS I data shows that the Navy is served reasonably well by present fleet prediction models. Of the 50 cases considered in this study with new surface duct measurements, the operational model AMOS and the laboratory Rough Surface Natural Mode Model compared approximately the same on 24 cases. In six cases, AMOS had a better fit with experimental data, and in the remaining 21 cases, the Natural Mode Model had a closer fit with the experimental data. Continuation of the work for two more years will confirm this judgment over a wider range of duct conditions as well as supplying the experimental data required for development of variable profile models.

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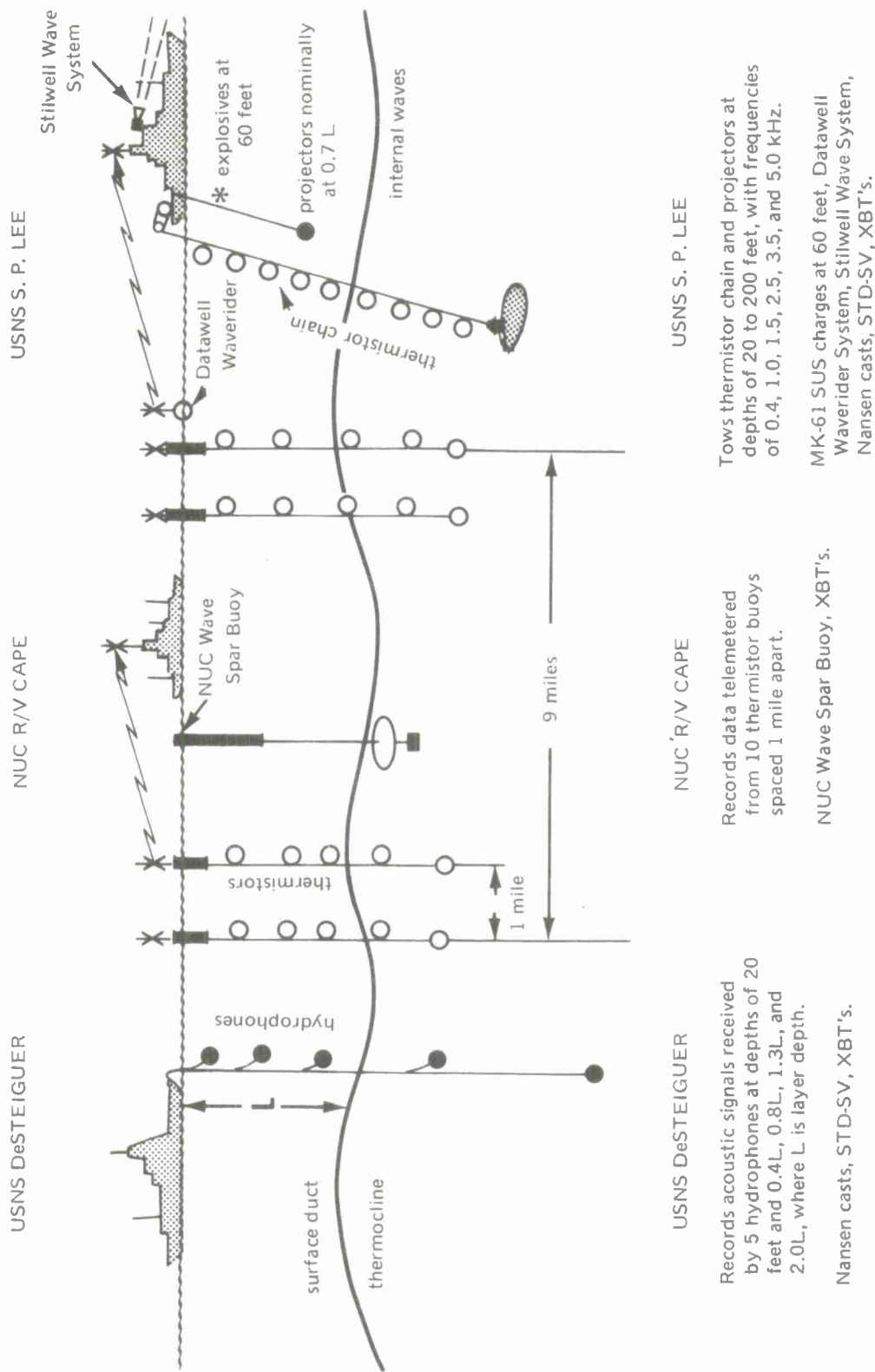


Figure 1. SUDS experiment.

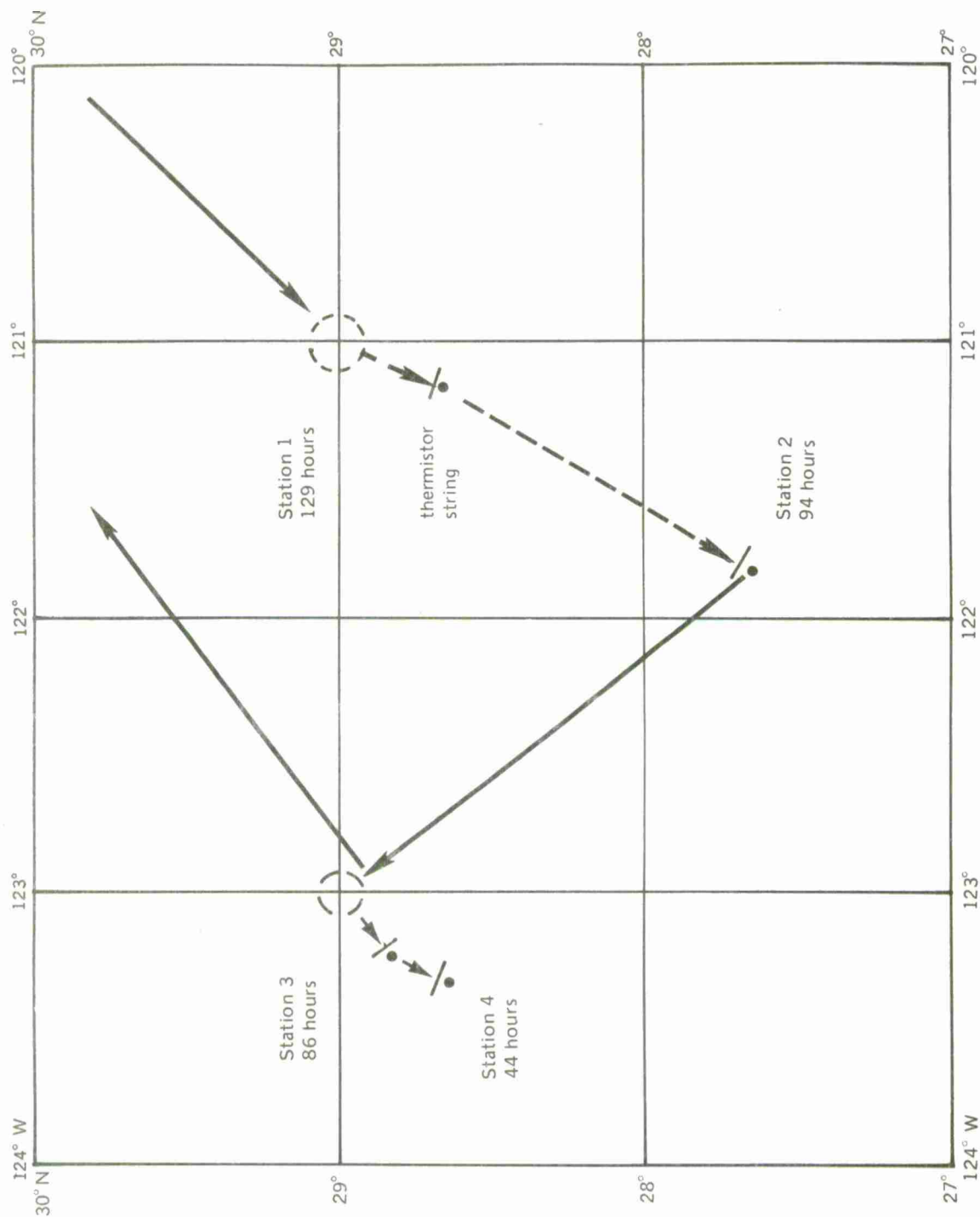
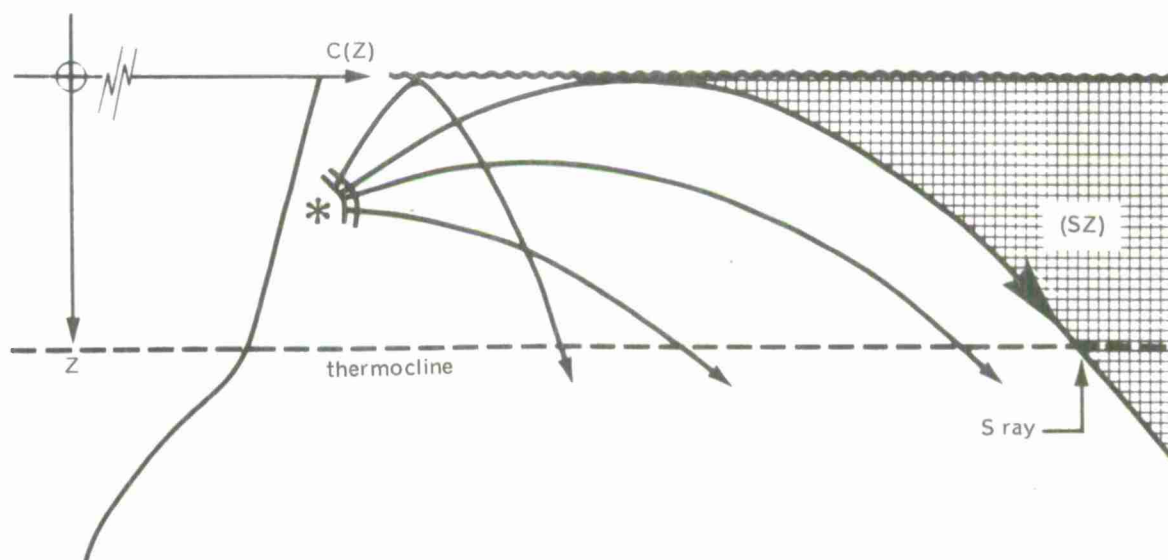
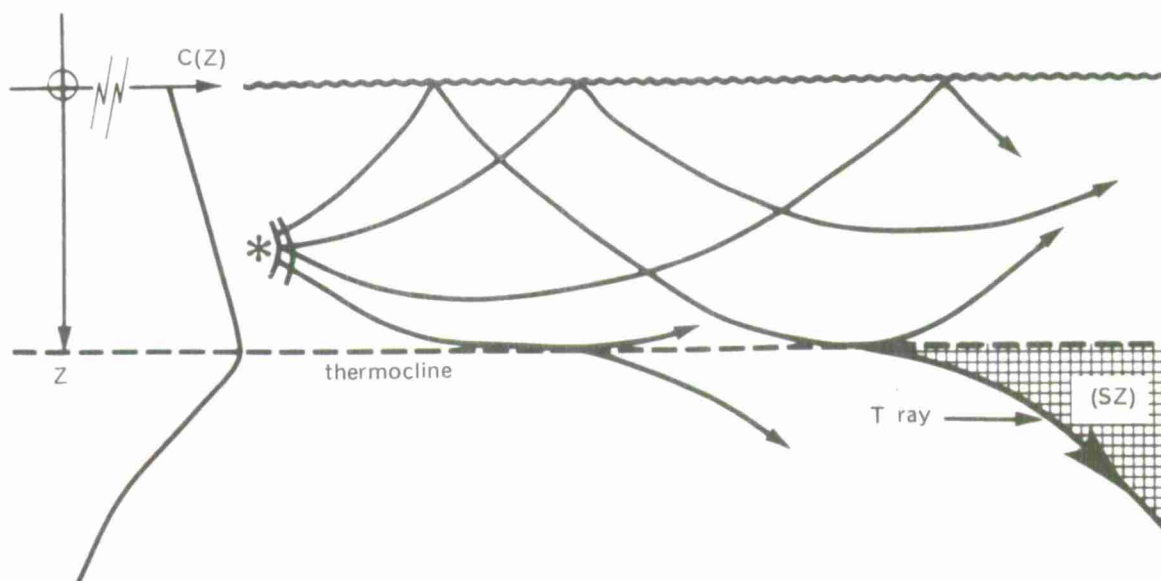


Figure 2. SUDS 1 track chart.





Part A. Negative sound-speed gradient in the mixed layer; no surface duct exists.



Part B. Positive sound-speed gradient in the mixed layer; surface duct exists.

Figure 3. Ray path examples.

Part A.

Station 2, Run 3

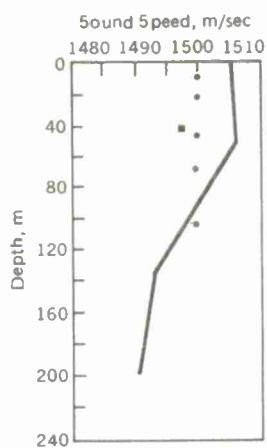
1328-1940, 15 Feb 72

Wind Speed: 10.3 m/sec

20 knots

Average Wave Height: 1.5 m

Depth, m	Sound Speed, m/sec
0	1505.37
50	1506.03
86	1499.59
134	1492.46
200	1489.68



Part B.

Station 3, Run 2

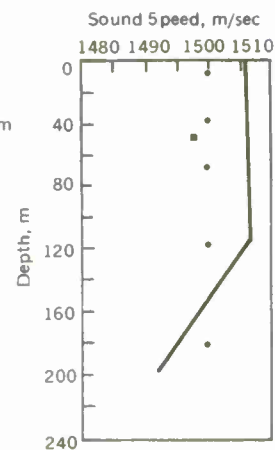
0105-0630, 20 Feb 72

Wind Speed: 4.1 m/sec

8 knots

Average Wave Height: 0.18 m

Depth, m	Sound Speed, m/sec
0	1505.78
112	1506.31
200	1491.74



Part C.

Station 3, Run 3

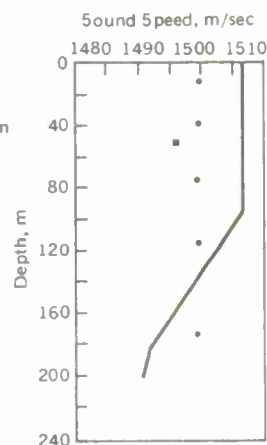
0658-1418, 20 Feb 72

Wind Speed: 5.2 m/sec

10 knots

Average Wave Height: 0.27 m

Depth, m	Sound Speed, m/sec
0	1506.09
90	1506.59
180	1492.34
200	1491.38



Part D.

Station 3, Run 4

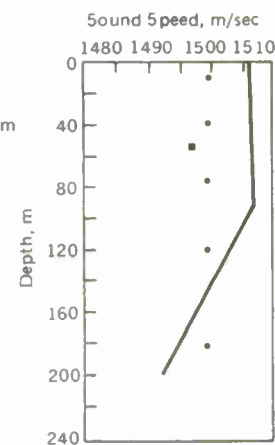
1531-2052, 20 Feb 72

Wind Speed: 4.1 m/sec

8 knots

Average Wave Height: 0.18 m

Depth, m	Sound Speed, m/sec
0	1506.09
90	1506.91
200	1492.11



Part E.

Station 4, Run 4

0116-0632, 23 Feb 72

Wind Speed: 5.2 m/sec

10 knots

Average Wave Height: 0.27 m

Depth, m	Sound Speed, m/sec
0	1507.13
93	1507.32
150	1497.62
200	1490.51

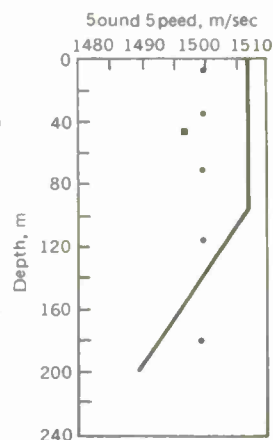
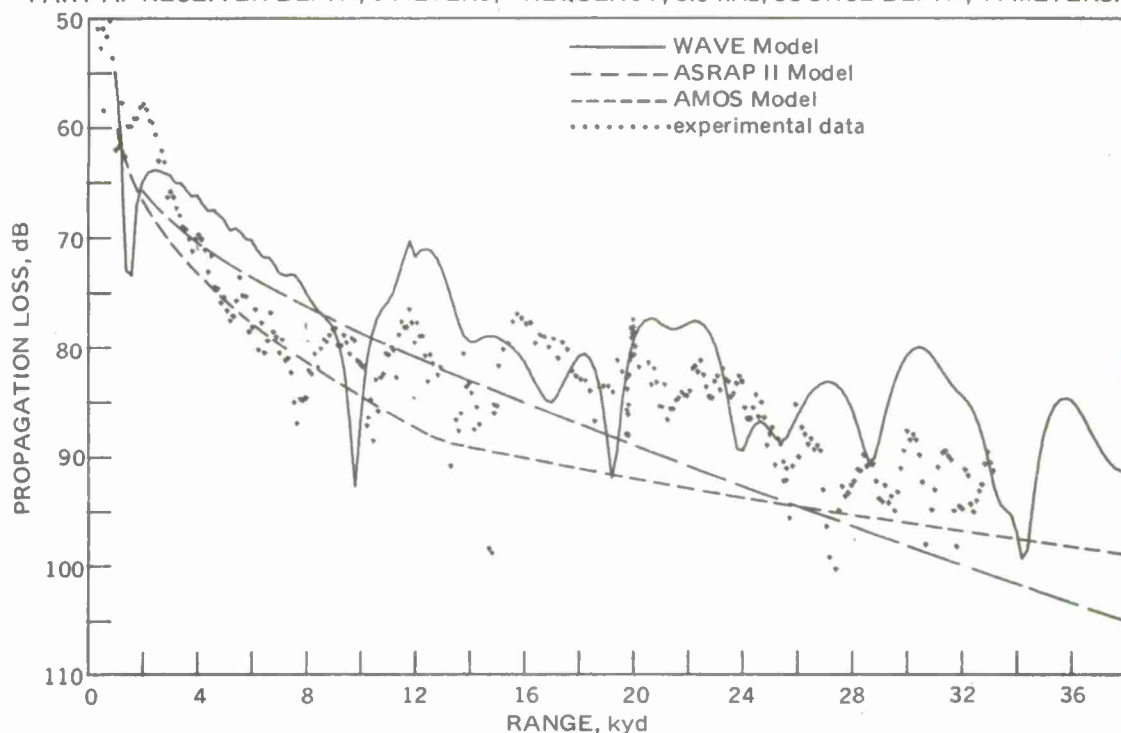


Figure 4. Sound-speed profiles for five selected runs. (The source is denoted by the symbol ■, and the receivers by the symbol ●.)

PART A. RECEIVER DEPTH, 6 METERS; FREQUENCY, 3.5 kHz; SOURCE DEPTH, 44 METERS.



PART B. RECEIVER DEPTH, 36 METERS; FREQUENCY, 3.5 kHz; SOURCE DEPTH, 44 METERS.

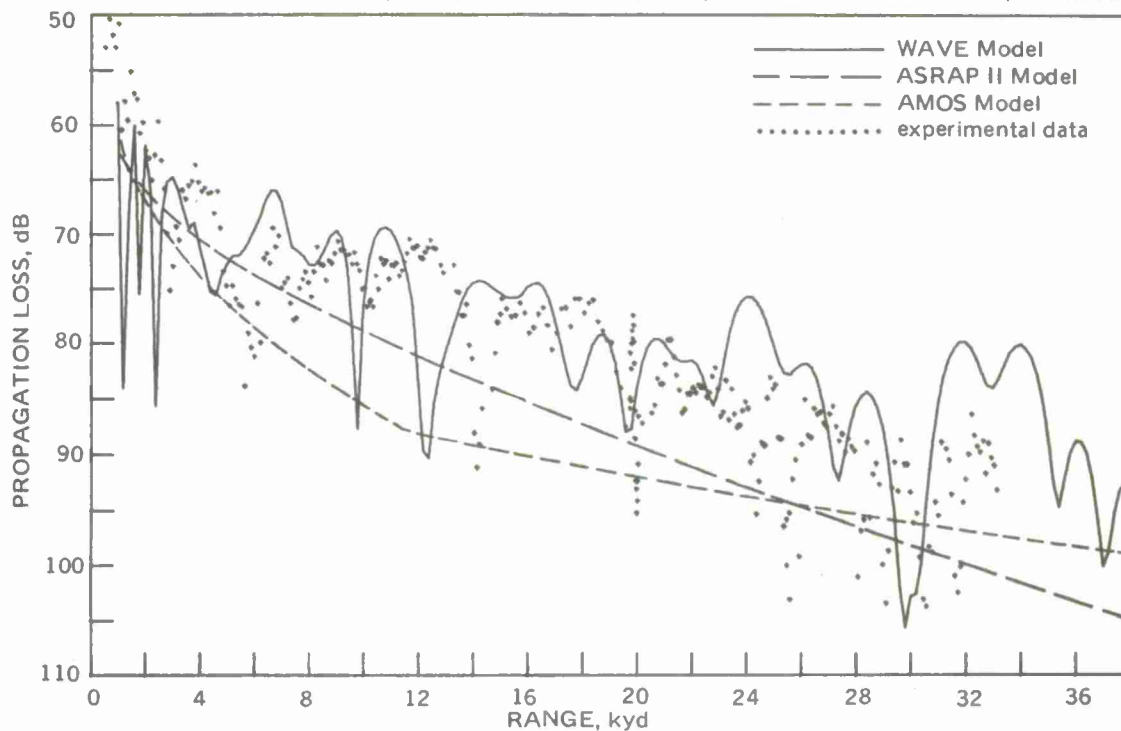
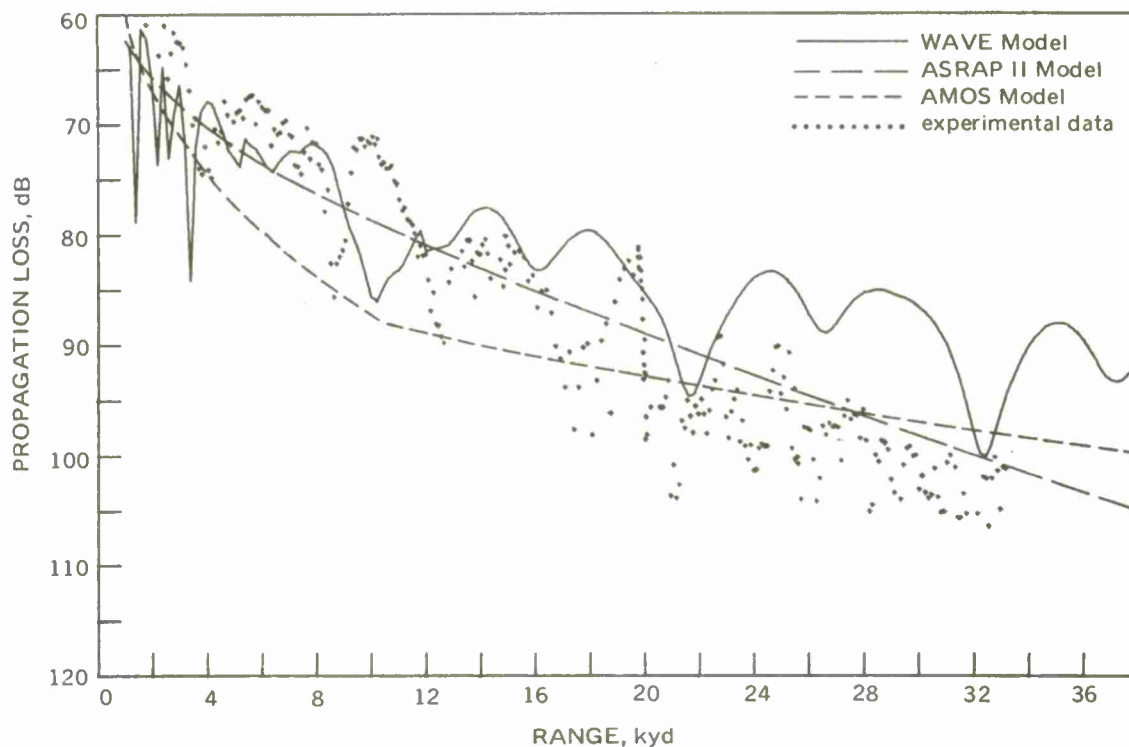


Figure 5. Propagation loss for AMOS, ASRAP II, and WAVE Models for Station 3, Run 4. 1531 to 2052 on 20 February 1972.

PART C. RECEIVER DEPTH, 72 METERS; FREQUENCY, 3.5 kHz; SOURCE DEPTH, 44 METERS.



PART D. RECEIVER DEPTH, 117 METERS; FREQUENCY, 3.5 kHz; SOURCE DEPTH, 44 METERS.

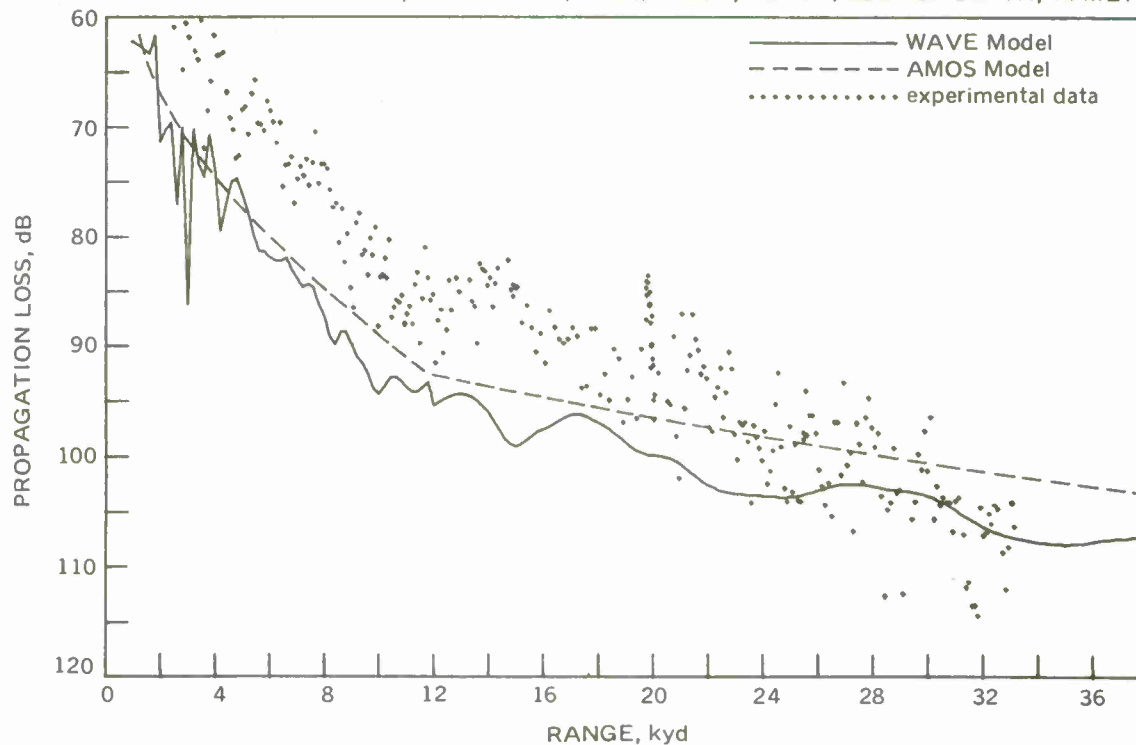
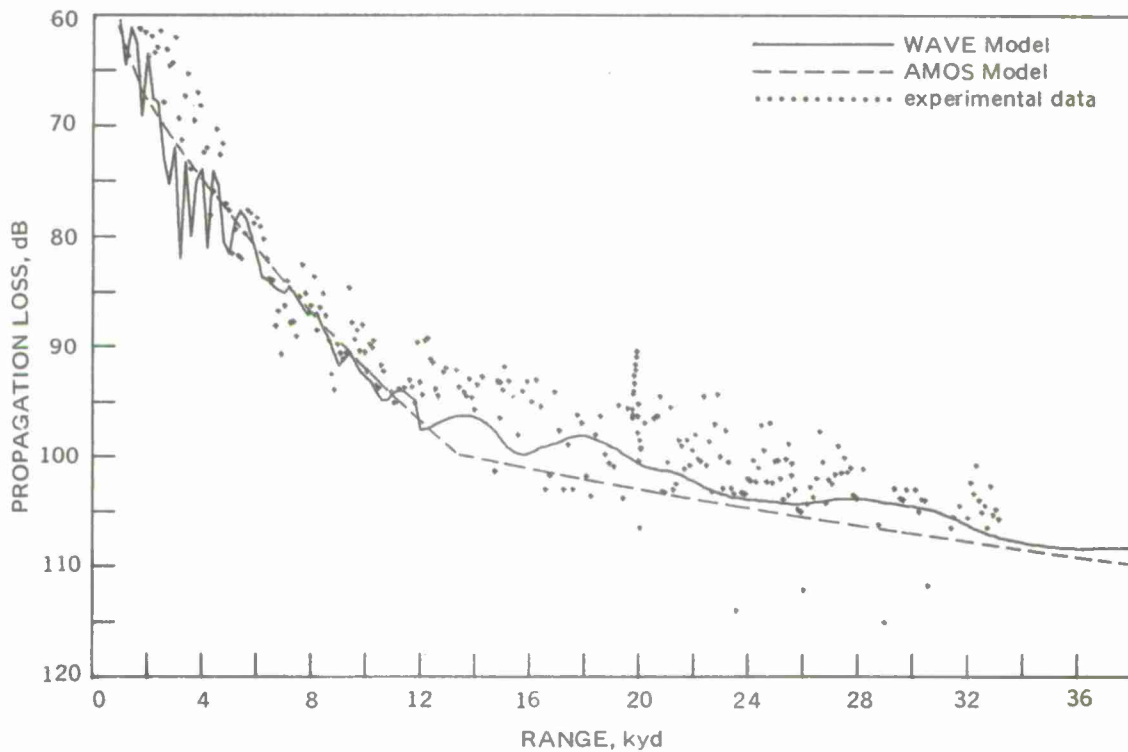


Figure 5. (Continued).

PART E. RECEIVER DEPTH, 180 METERS; FREQUENCY, 3.5 kHz; SOURCE DEPTH, 44 METERS.



PART F. RECEIVER DEPTH, 36 METERS; FREQUENCY, 5.0 kHz; SOURCE DEPTH, 47 METERS.

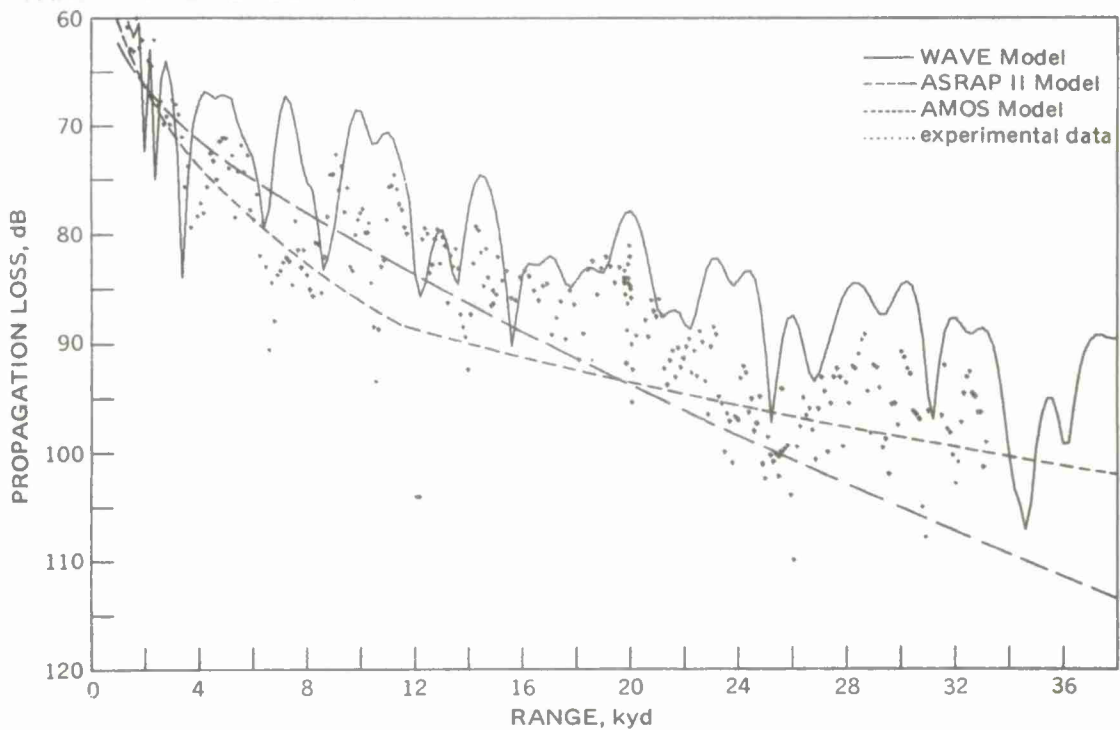


Figure 5. (Continued).

PART G. RECEIVER DEPTH, 117 METERS; FREQUENCY, 5.0 kHz; SOURCE DEPTH, 47 METERS.

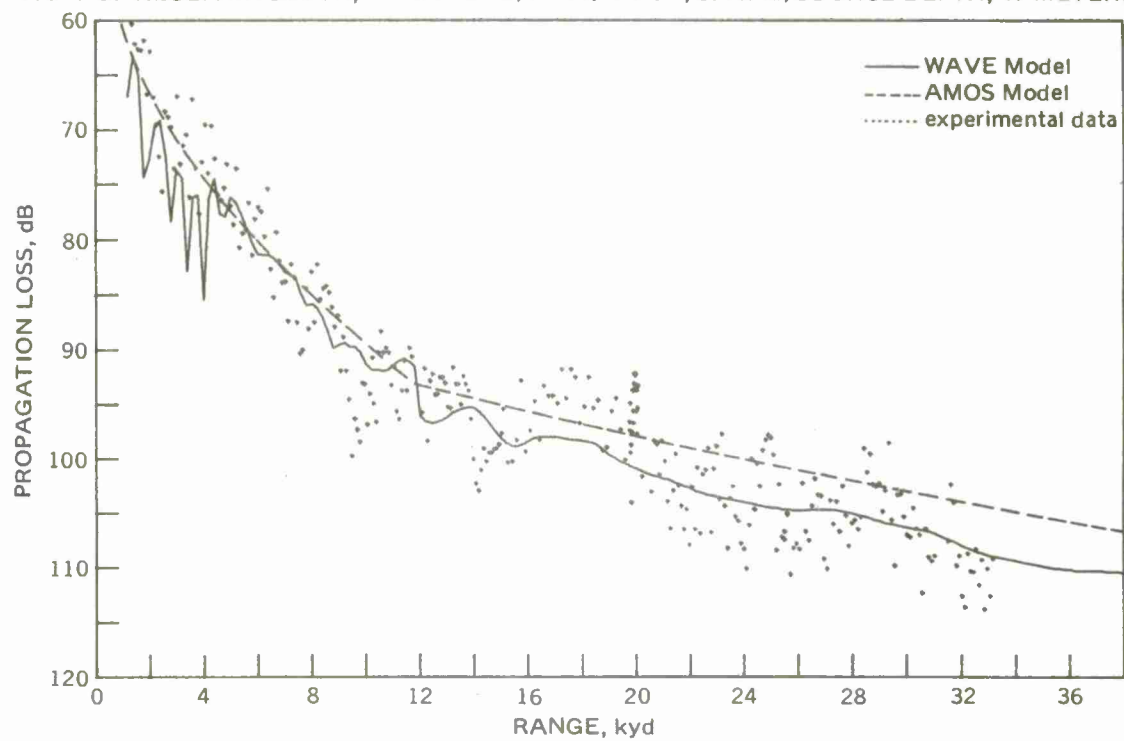
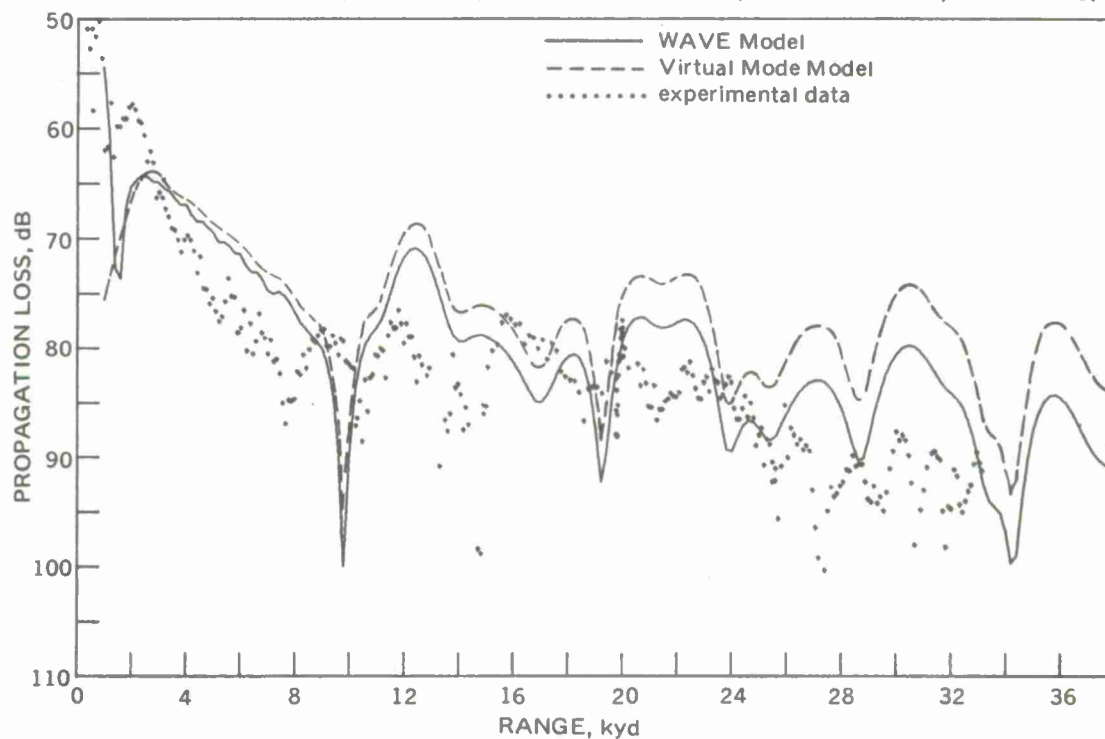


Figure 5. (Continued).

PART A. RECEIVER DEPTH, 6 METERS; FREQUENCY 3.5 kHz; SOURCE DEPTH, 44 METERS.



PART B. RECEIVER DEPTH, 36 METERS; FREQUENCY, 3.5 kHz; SOURCE DEPTH, 44 METERS.

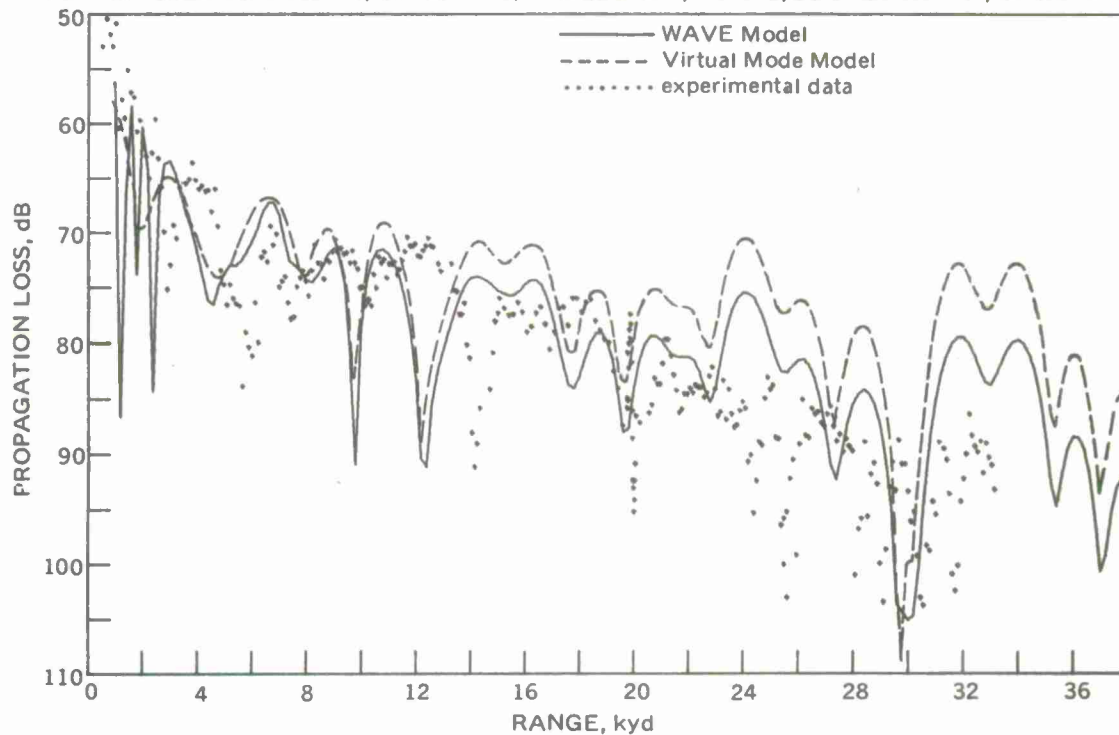
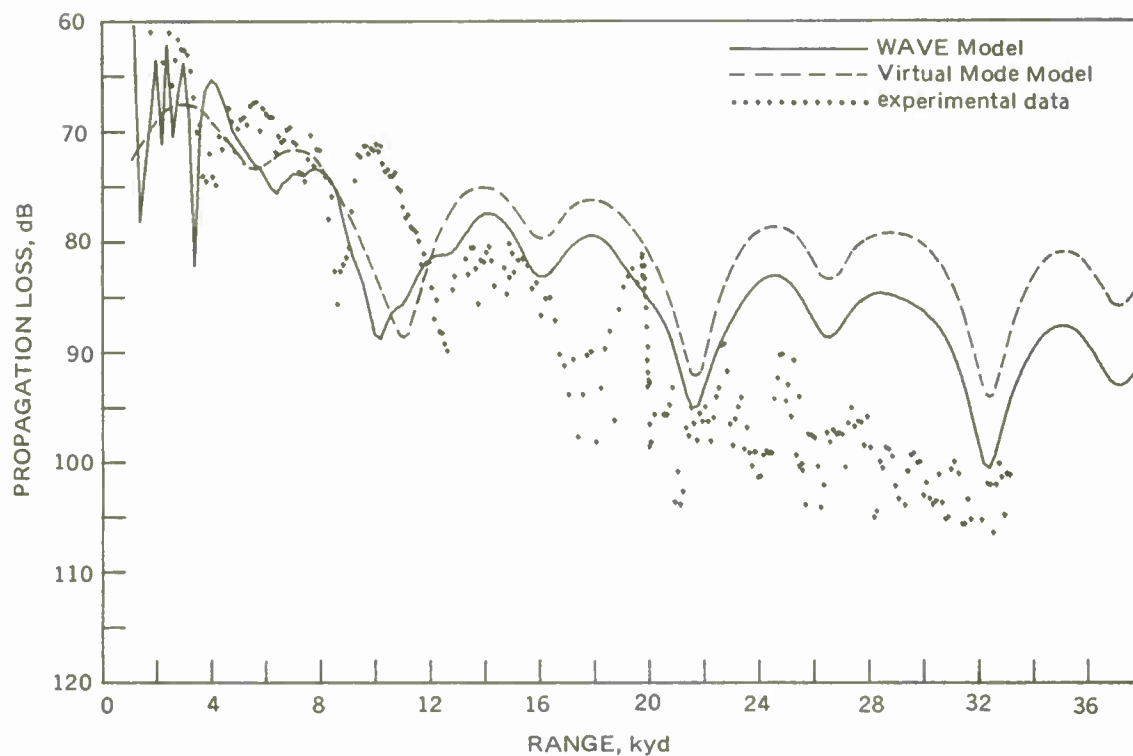


Figure 6. Comparisons of the Virtual Mode and WAVE Models, both without surface scattering corrections, for Station 3, Run 4. 1531 to 2052 on 20 February 1972.



PART C. RECEIVER DEPTH, 72 METERS; FREQUENCY, 3.5 kHz; SOURCE DEPTH, 44 METERS.



PART D. RECEIVER DEPTH, 117 METERS; FREQUENCY, 3.5 kHz; SOURCE DEPTH, 44 METERS.

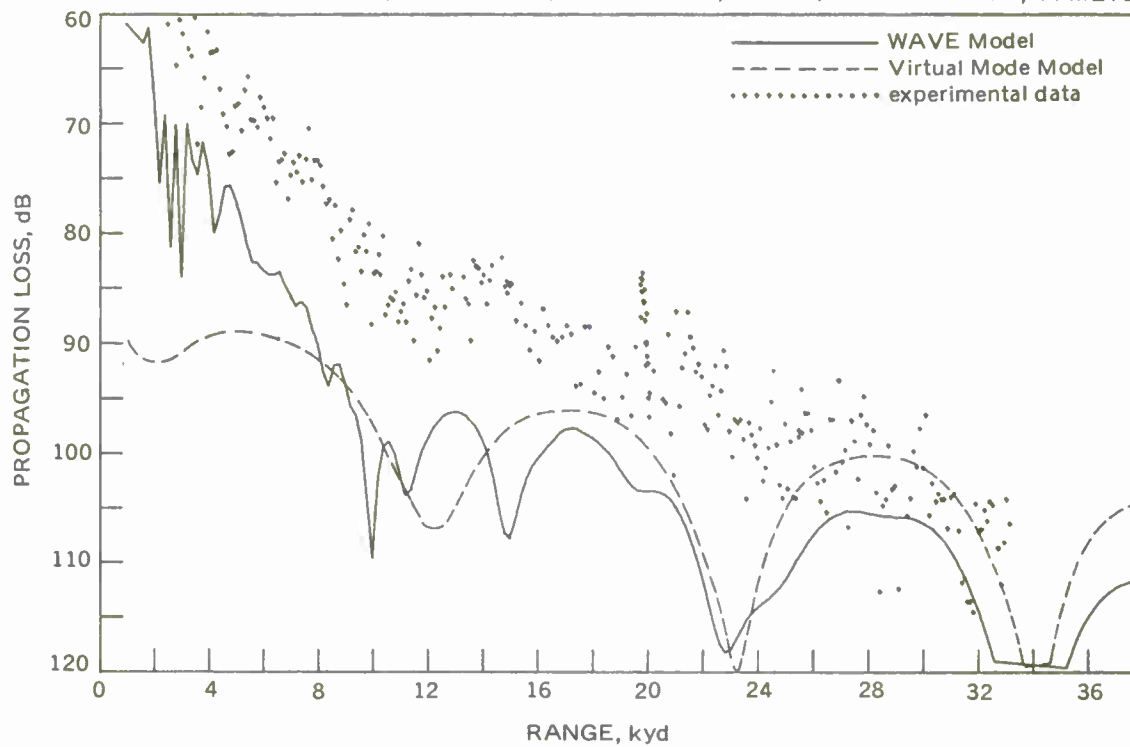


Figure 6. (Continued).

PART E. RECEIVER DEPTH, 180 METERS; FREQUENCY, 3.5 kHz; SOURCE DEPTH, 44 METERS.

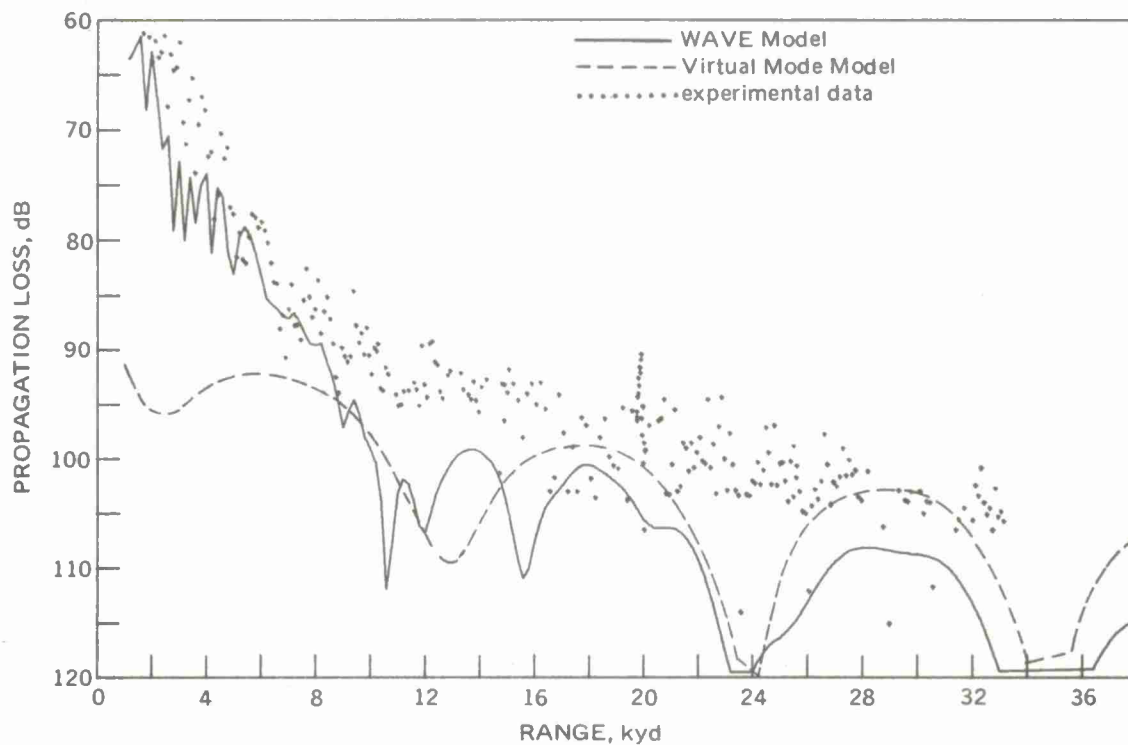


Figure 6. (Continued).

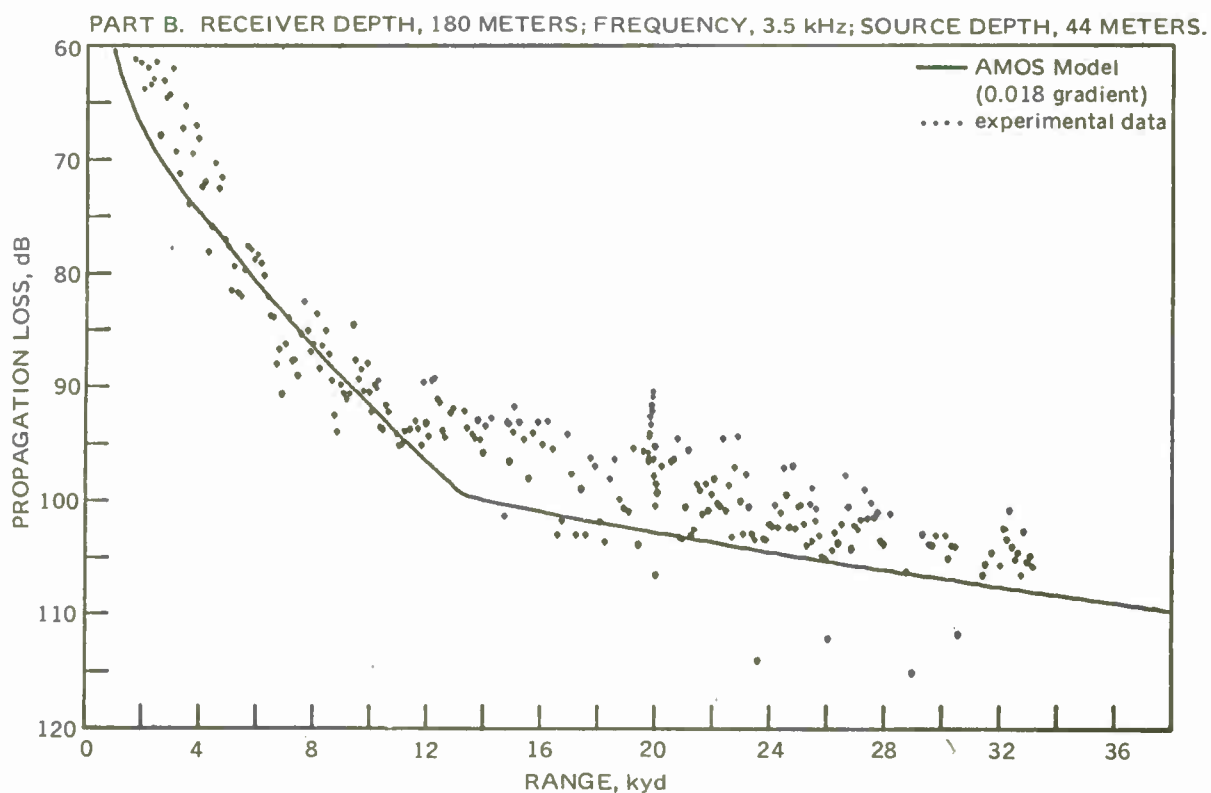
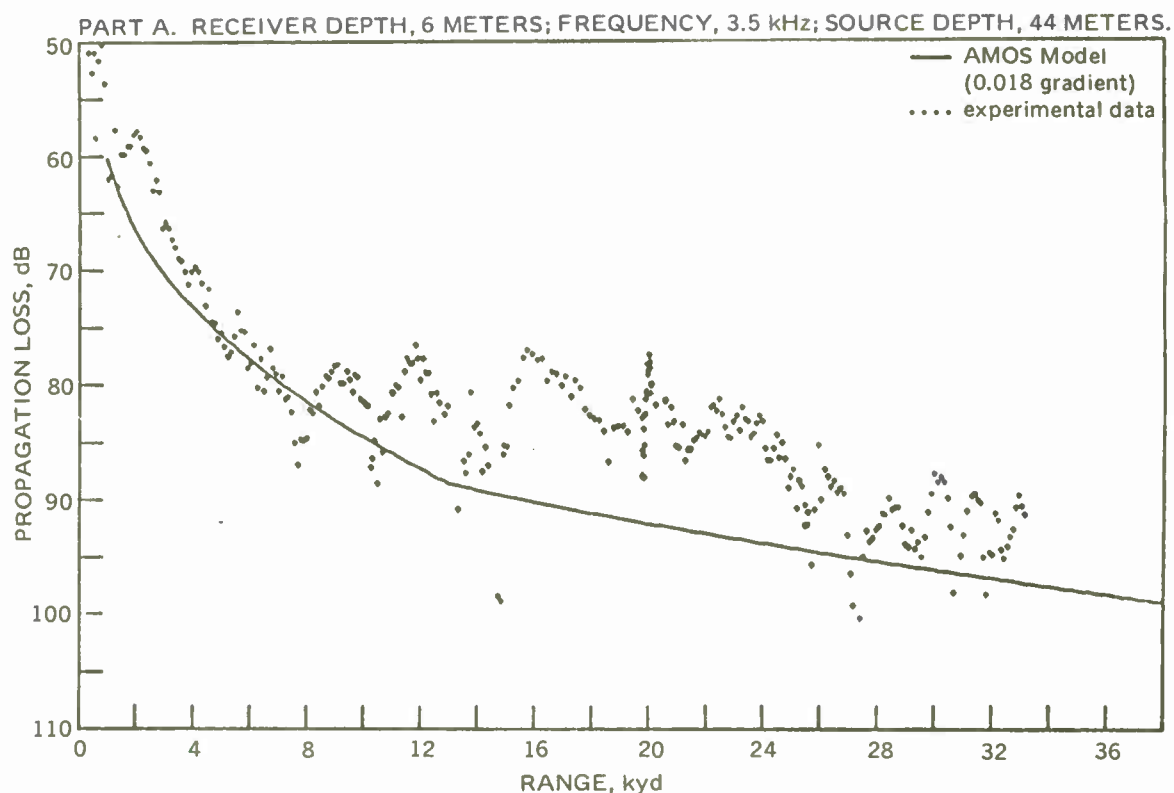
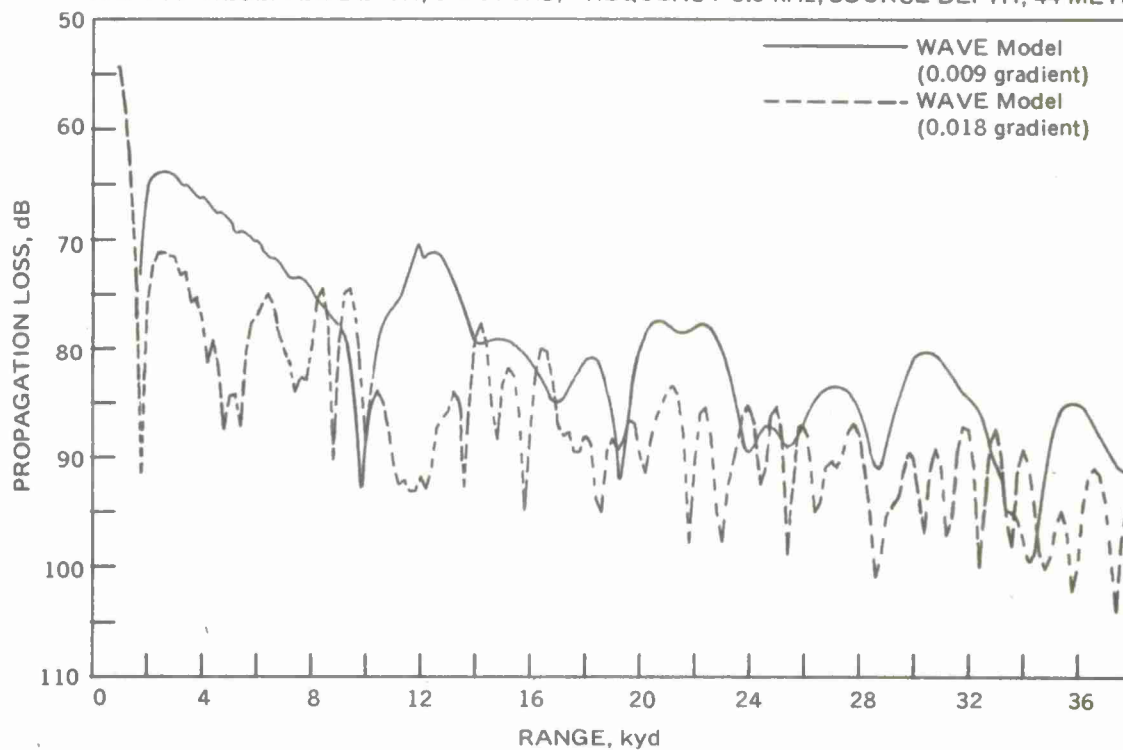


Figure 7. Comparison of propagation loss for the AMOS Model using an assumed gradient equal to 0.018. Station 3, Run 4. 1531 to 2052 on 20 February 1972.

PART A. RECEIVER DEPTH, 6 METERS; FREQUENCY 3.5 kHz; SOURCE DEPTH, 44 METERS.



PART B. RECEIVER DEPTH, 117 METERS; FREQUENCY, 3.5 kHz; SOURCE DEPTH, 44 METERS.

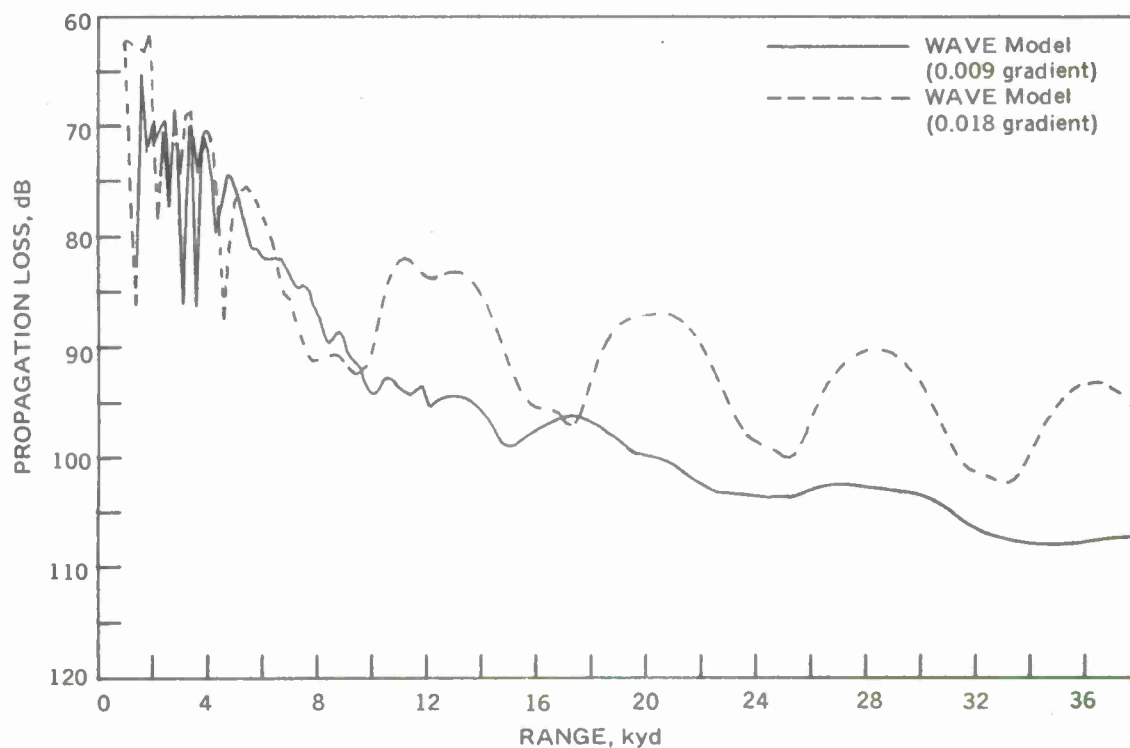


Figure 8. Comparison of propagation loss from the WAVE Model for gradients equal to 0.009 and 0.018. Station 3, Run 4. 1531 to 2052 on 20 February 1972.

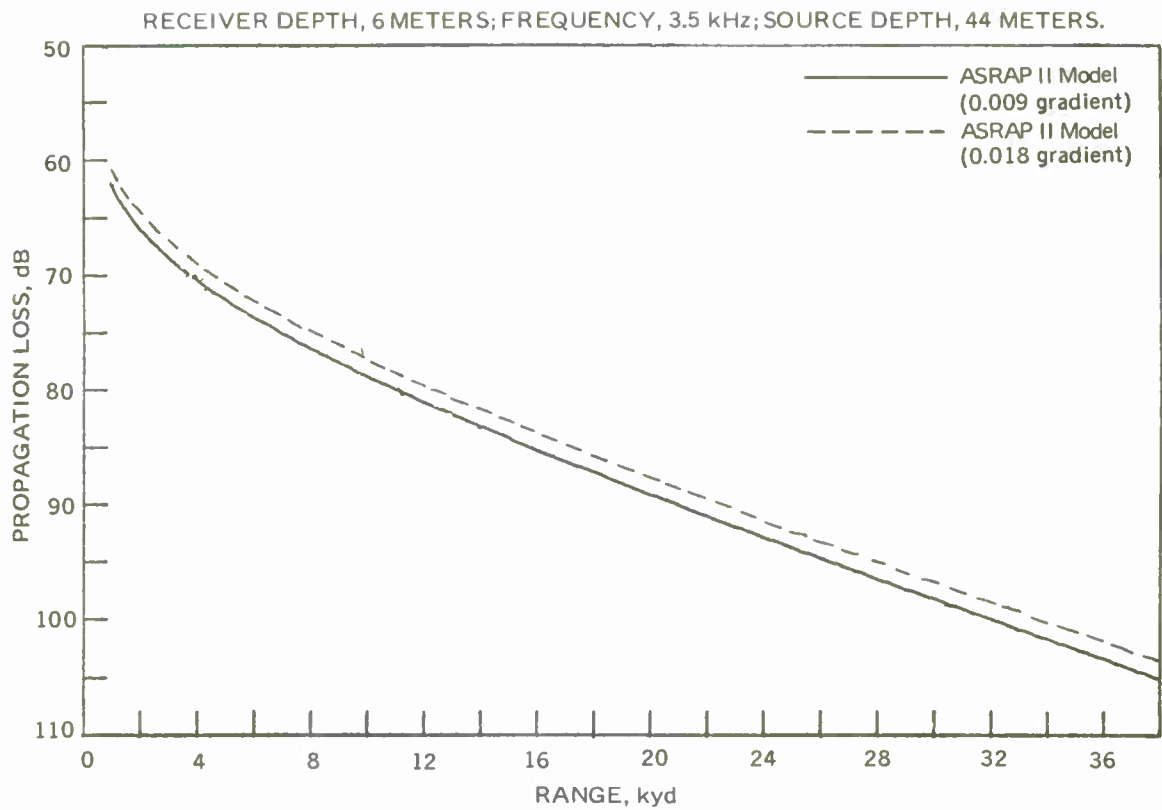
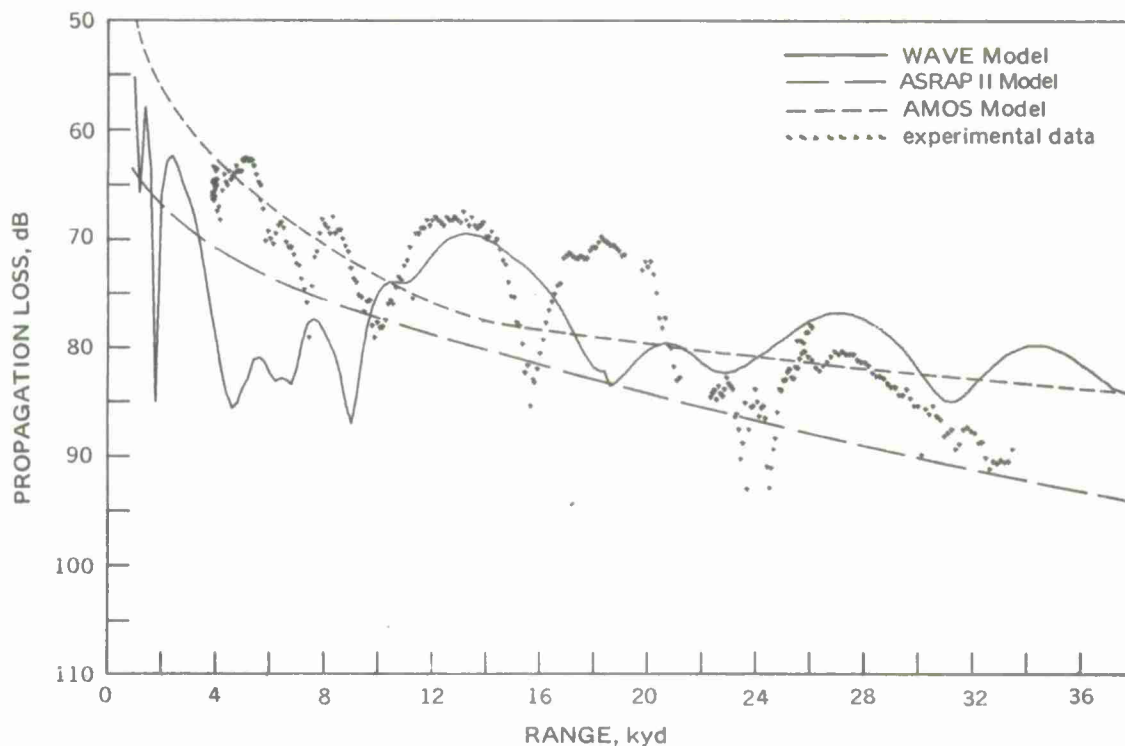


Figure 9. Comparison of propagation loss from the ASRAP II Model for gradients equal to 0.009 and 0.018. Station 3, Run 4. 1531 to 2052 on 20 February 1972.

PART A. RECEIVER DEPTH, 37 METERS; FREQUENCY, 1.5 kHz; SOURCE DEPTH, 42 METERS



PART B. RECEIVER DEPTH, 73 METERS; FREQUENCY, 2.5 kHz; SOURCE DEPTH, 42 METERS

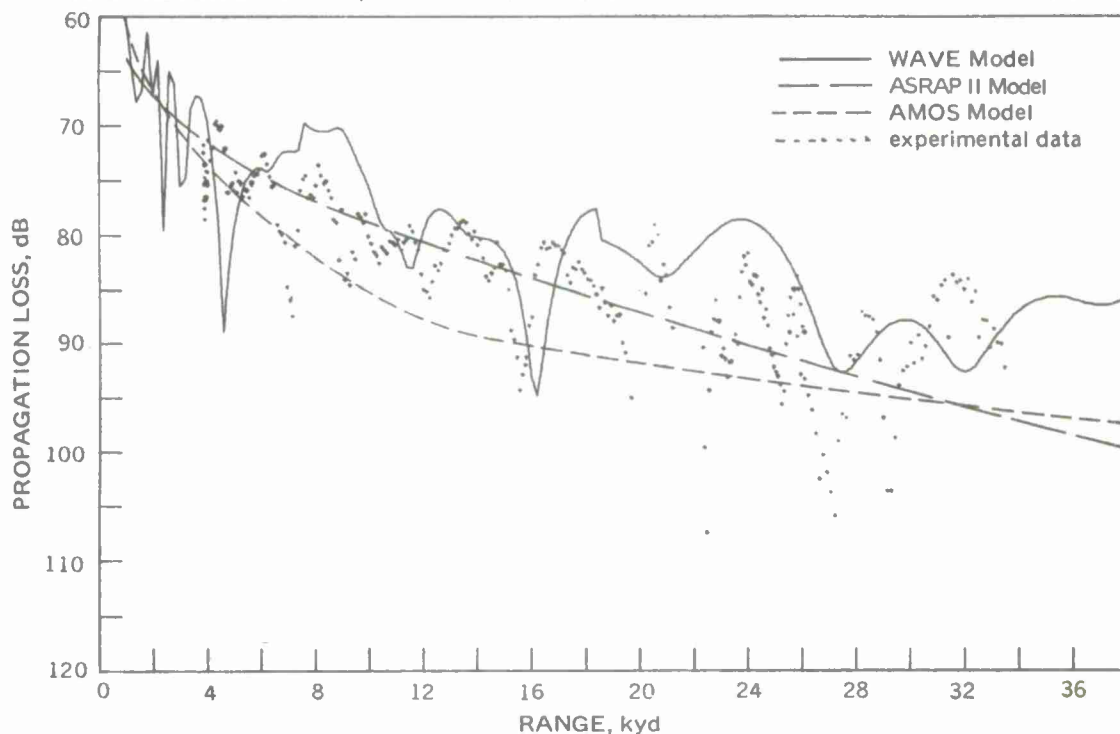
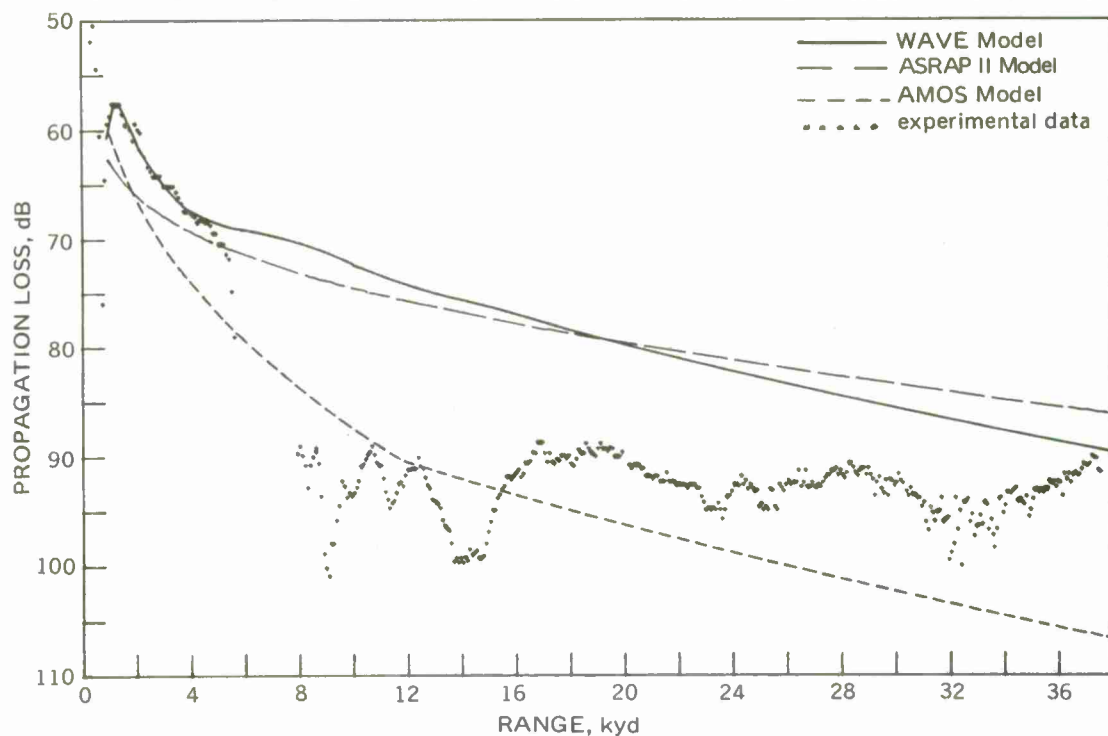


Figure 10. Comparison of propagation loss from theoretical models WAVE, ASRAP II, and AMOS. Station 3, Run 2. 0105 to 0630 on 20 February 1972.

PART A. RECEIVER DEPTH, 34 METERS; FREQUENCY, 0.4 kHz; SOURCE DEPTH, 34 METERS.



PART B. RECEIVER DEPTH, 112 METERS; FREQUENCY, 0.4 kHz; SOURCE DEPTH, 42 METERS.

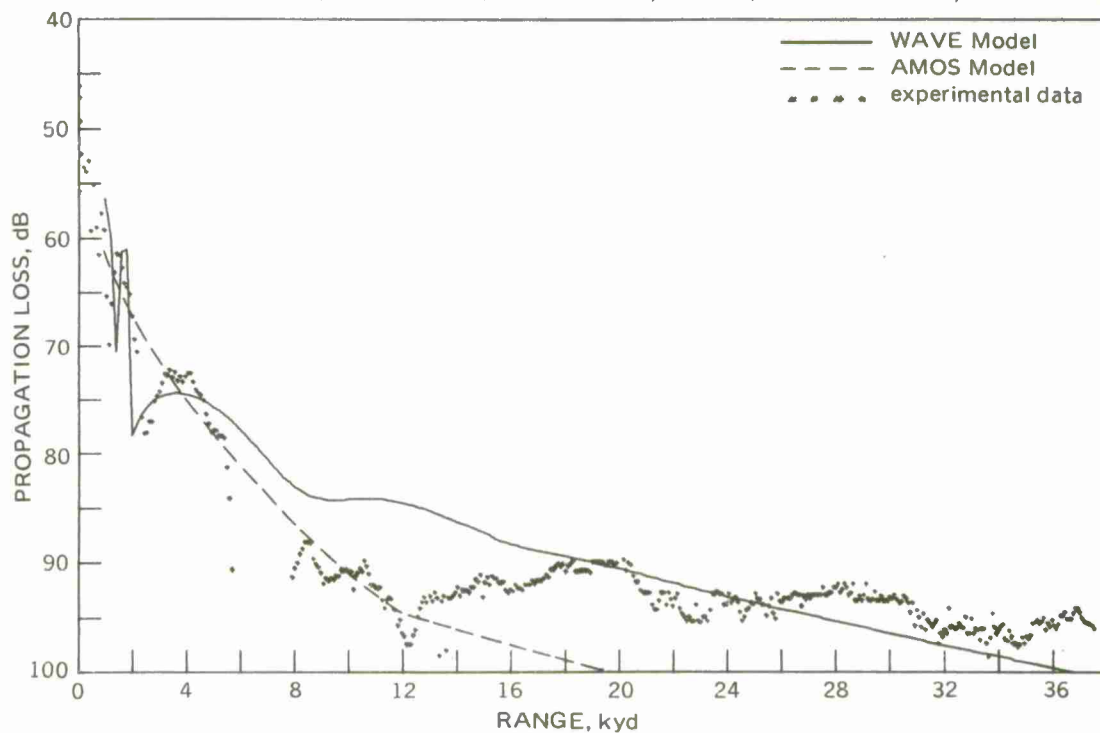


Figure 11. Comparison of propagation loss for Station 3, Run 3. 0658 to 1418 hours on 20 February 1972.



PART C. RECEIVER DEPTH, 6 METERS; FREQUENCY, 1.0 kHz; SOURCE DEPTH, 42 METERS.

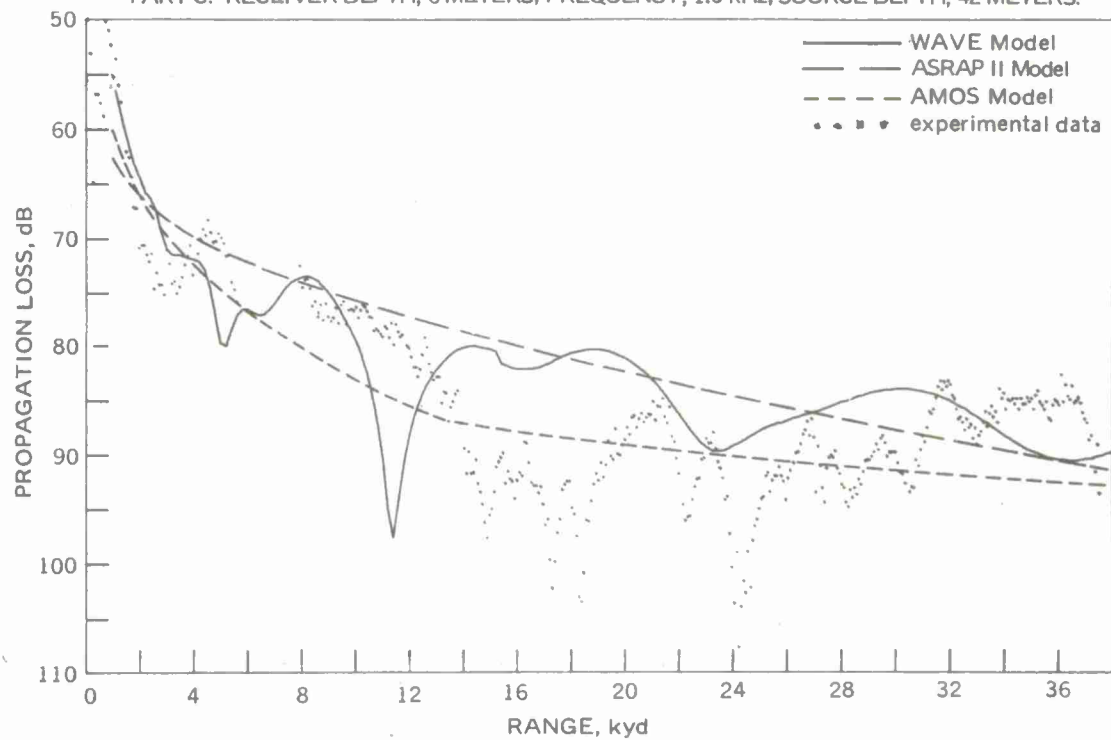
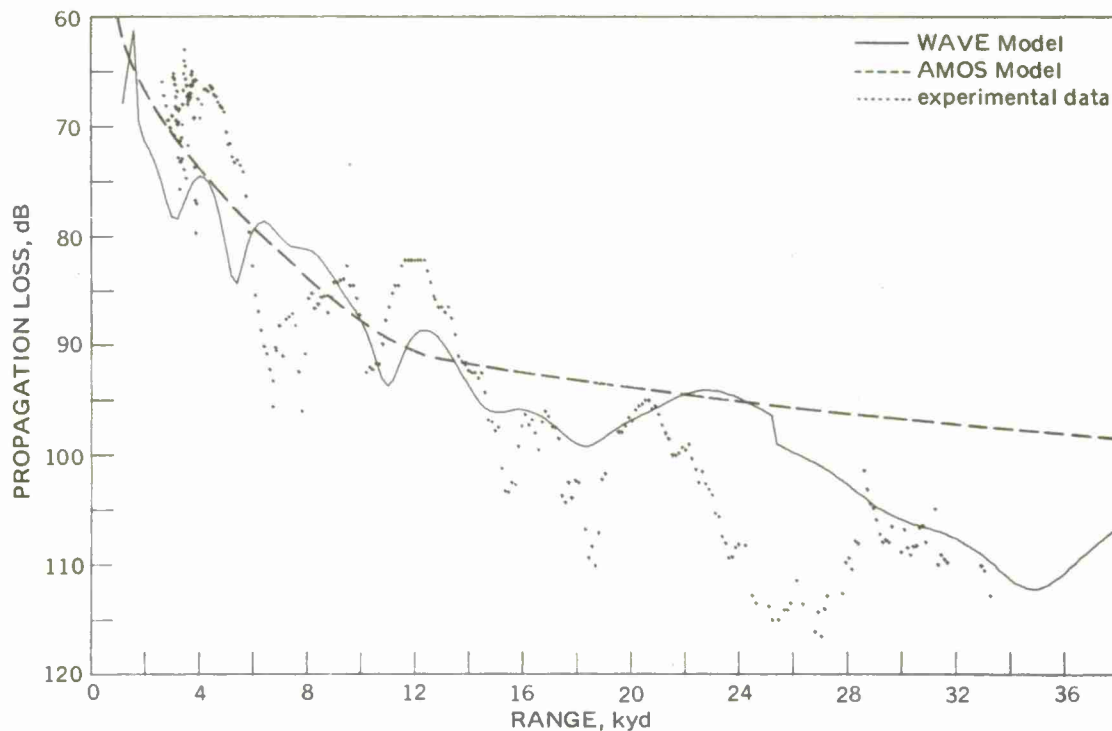


Figure 11. (Continued).

PART A. RECEIVER DEPTH, 117 METERS; FREQUENCY, 1.5 kHz; SOURCE DEPTH, 43 METERS.



PART B. RECEIVER DEPTH, 180 METERS; FREQUENCY 2.5 kHz; SOURCE DEPTH, 43 METERS.

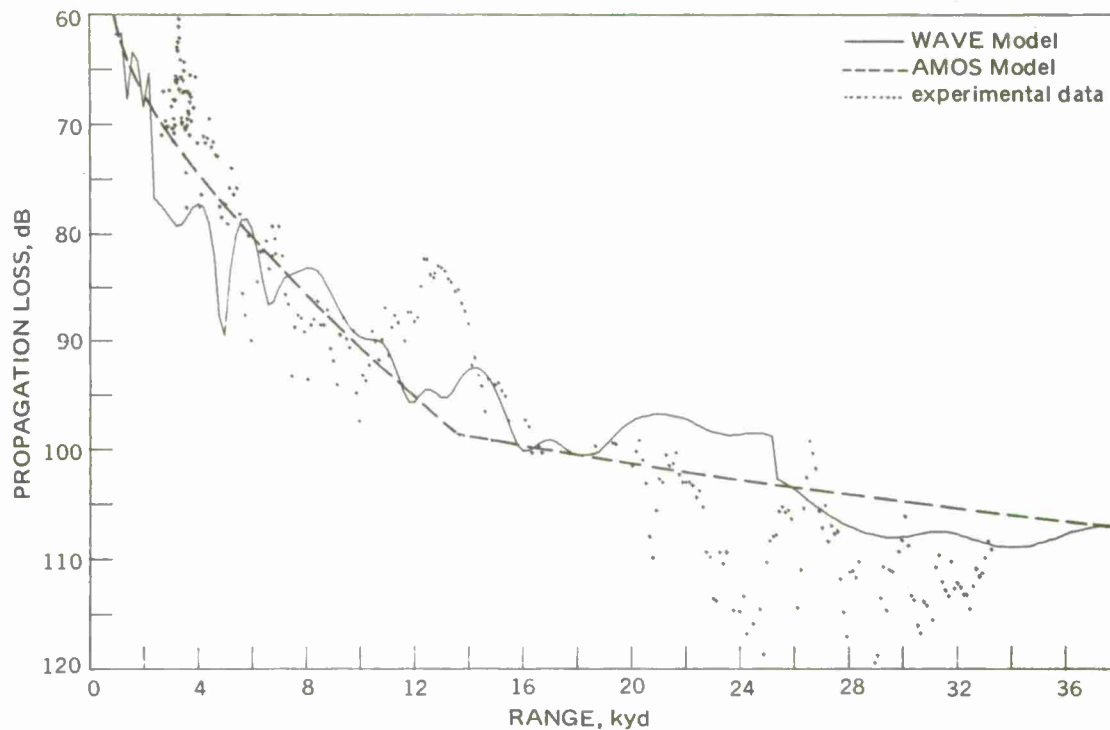
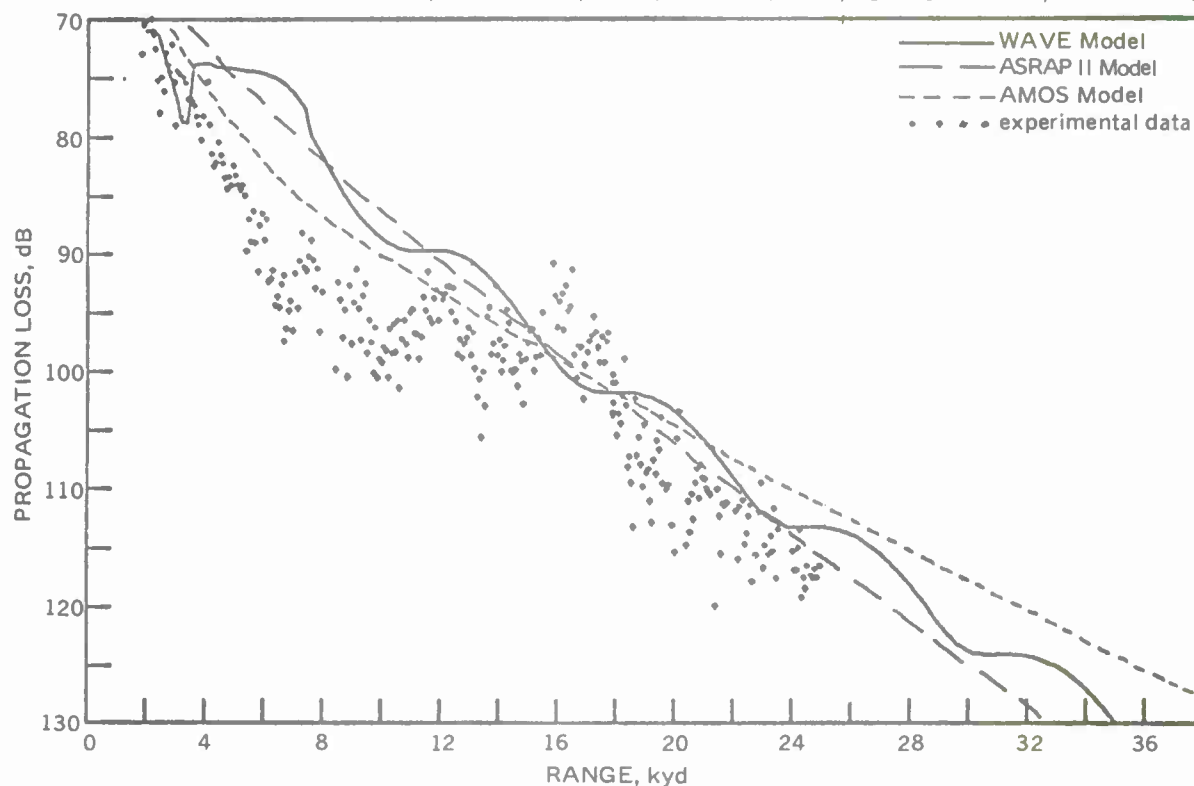


Figure 12. Comparison of propagation loss for Station 4, Run 4. 0116 to 0632 on 23 February 1972.

PART A. RECEIVER DEPTH, 43 METERS; FREQUENCY 3.5 kHz; SOURCE DEPTH, 38 METERS.



PART B. RECEIVER DEPTH, 72 METERS; FREQUENCY 3.5 kHz; SOURCE DEPTH, 38 METERS.

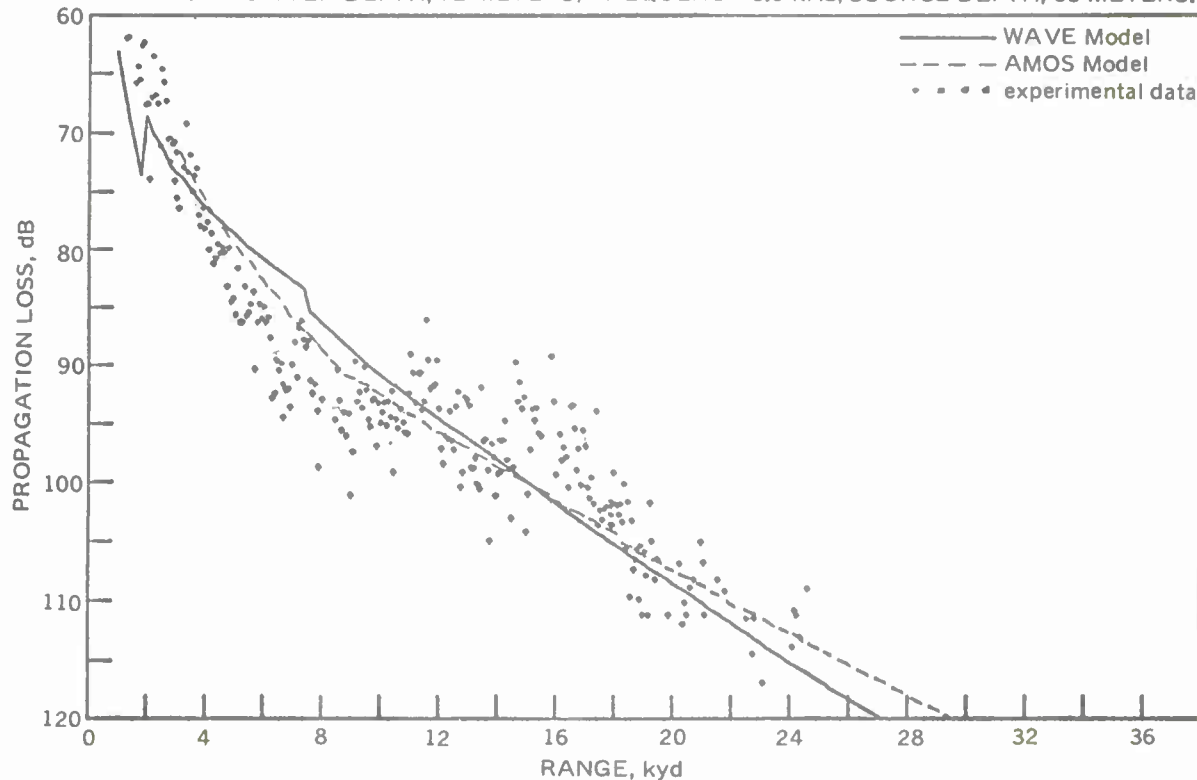
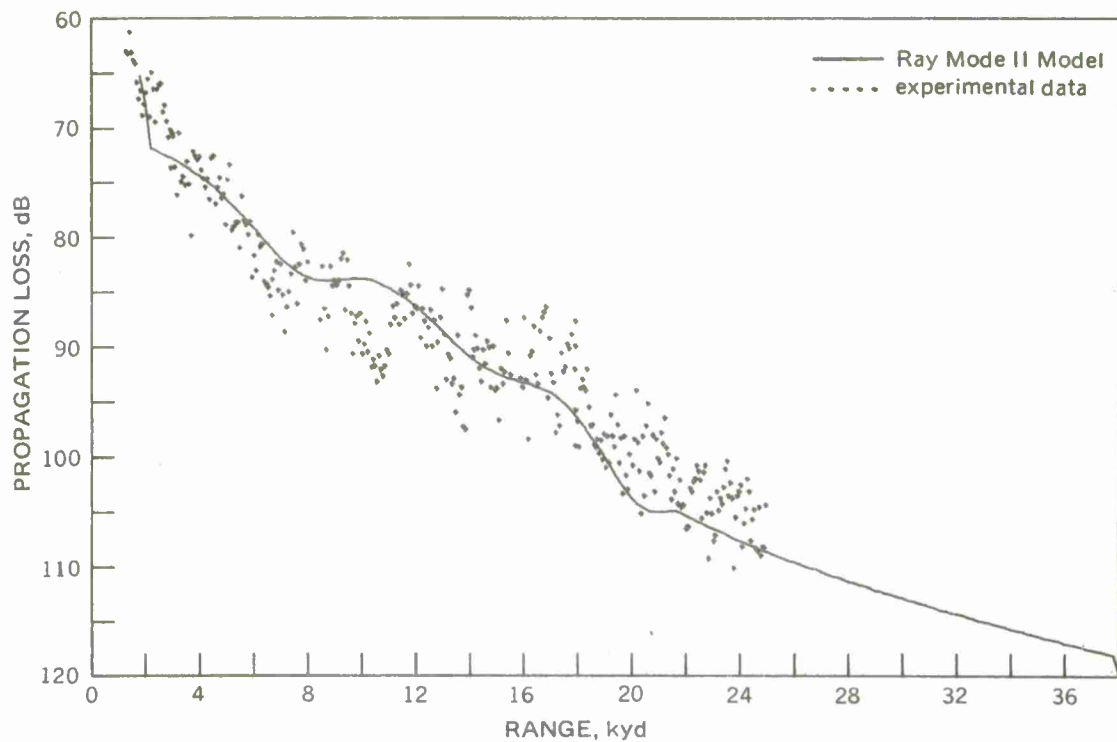


Figure 13. Comparison of propagation loss for Station 2, Run 3. 1328 to 1940 on 15 February 1972.

PART A. RECEIVER DEPTH, 4 METERS; FREQUENCY, 3.5 kHz; SOURCE DEPTH, 38 METERS.



PART B. RECEIVER DEPTH, 72 METERS; FREQUENCY, 3.5 kHz; SOURCE DEPTH, 38 METERS.

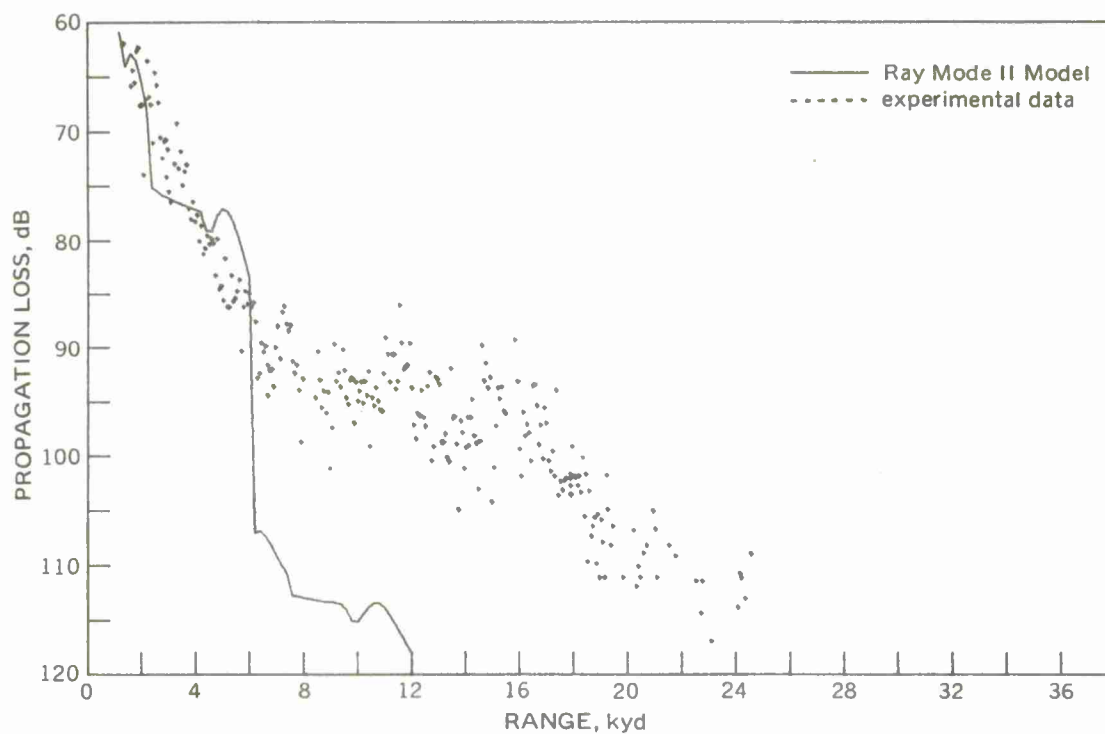
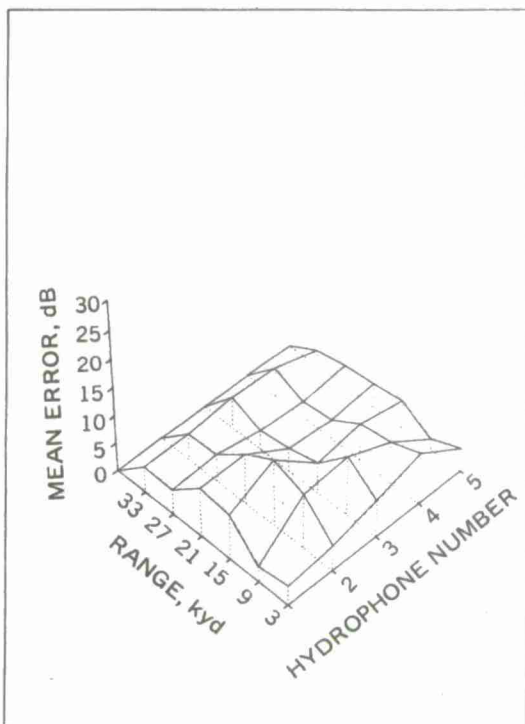
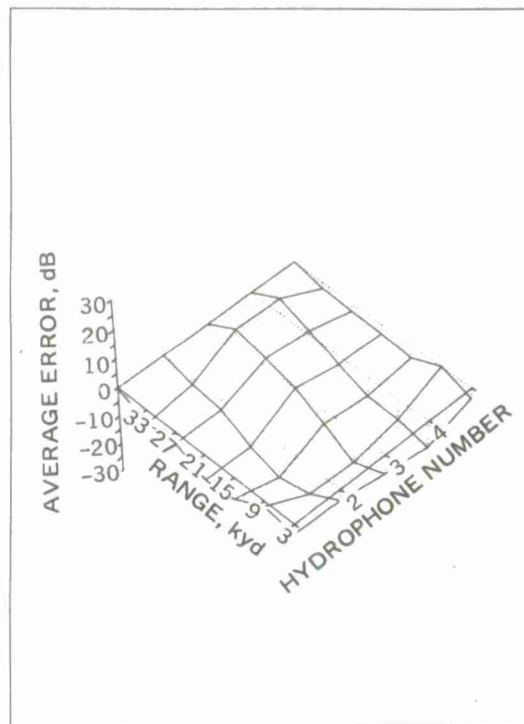


Figure 14. Comparison of propagation loss for Station 2, Run 3. 1328 to 1940 on 15 February 1972.

PART A. MEAN ERROR SURFACE.



PART B. AVERAGE ERROR SURFACE.



PART C. QUADRATIC DIFFERENCE SURFACE. PART D. LINEAR CORRELATION SURFACE.

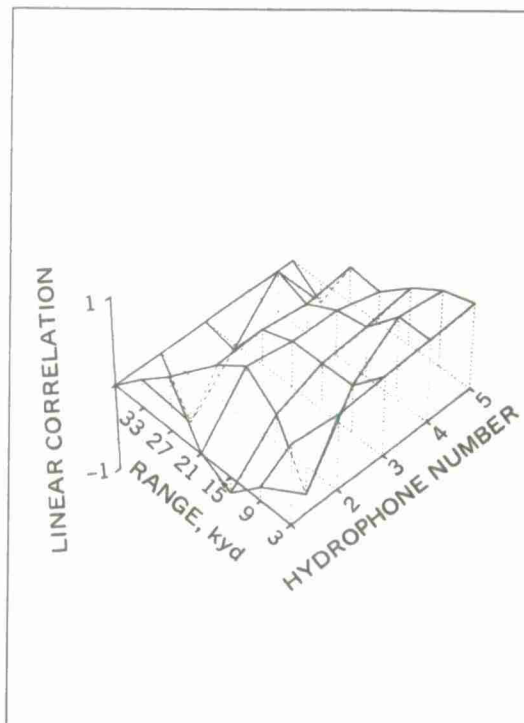
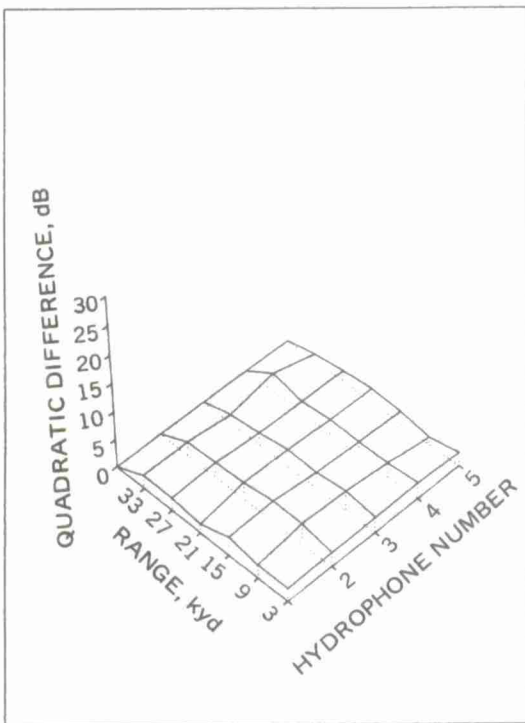
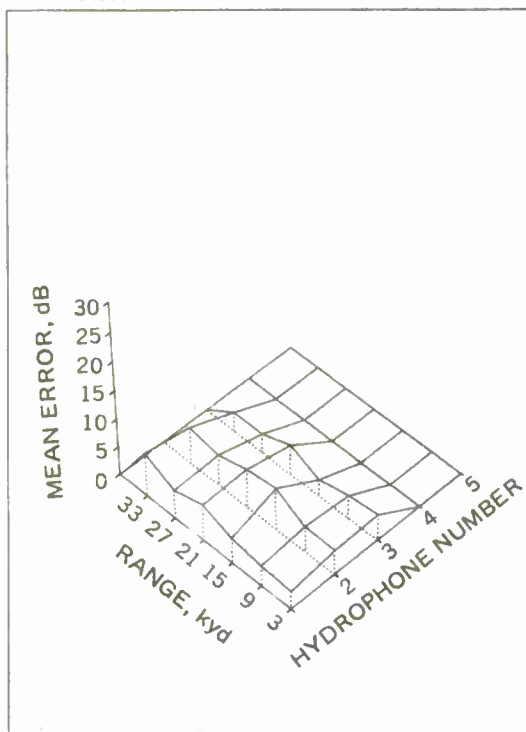
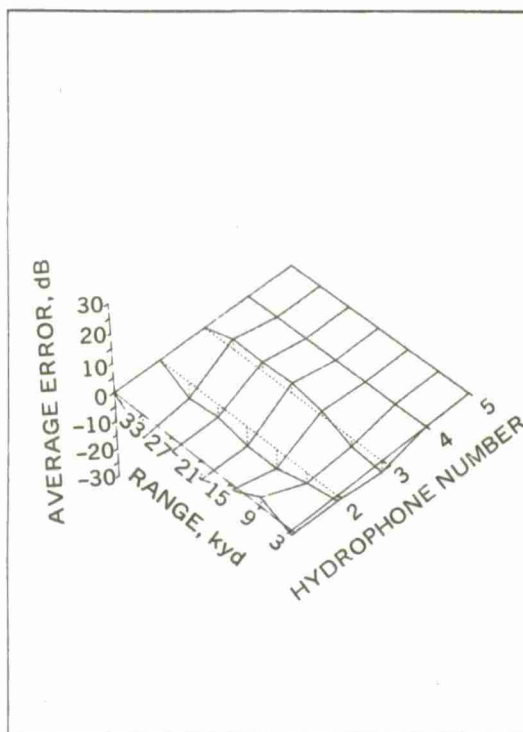


Figure 15. Mean error, average error, quadratic difference, and linear correlation surfaces for the AMOS Model on Station 3, Run 4. 3.5-kHz frequency.

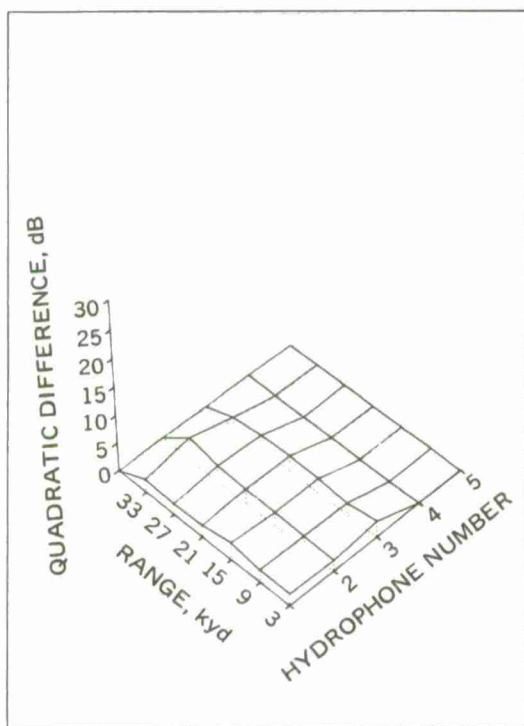
PART A. MEAN ERROR SURFACE.



PART B. AVERAGE ERROR SURFACE.



PART C. QUADRATIC DIFFERENCE SURFACE.



PART D. LINEAR CORRELATION SURFACE.

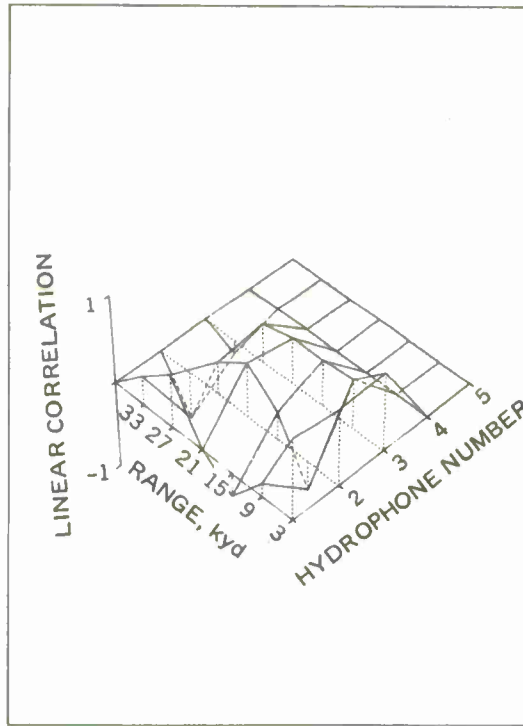


Figure 16. Mean error, average error, quadratic difference, and linear correlation surfaces for the ASRAP II Model on Station 3, Run 4. 3.5-kHz frequency.





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